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Some Results on Symmetric Varieties

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Let G be a connected reductive algebraic group defined over a field k of characteristic not 2, θ an involution of G defined over k and H a k-open subgroup of the fixed point group of θ. In this contribution we discuss a number of recent results about algebraic groups with involutions and the corresponding symmetric varieties G_k/H_k and G/H. We emphasize the cases where k is the real numbers or a non-archimedean local field.

The case $k = \mathbb{R}$ is well known and the representation theory of Riemannian and affine symmetric spaces has been studied extensively. More recently some mathematicians have started to study the cases of local fields and finite fields.

The main part of this talk concerns a dicussion of the orbits of symmetric varieties under the action of parabolic k-subgroups. These orbits are of importance, for example, in the study of discrete series representation of affine symmetric spaces. The topics to be discussed include a characterization of the orbits, open and closed orbits, orbit closures, a Bruhat order and also a partial classification for real reductive groups. Some of these results are also new for the case of real reductive groups.