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-Nonsplit extensions of Lie algebras and their irreducible representations-

Let $\kappa(L)$ is a number of irreducible modules of finite-dimensional Lie algebra L over a field P of characteristic $p \geq 0$ with nonsplit extensions.

THEOREM 1 ($p > 0$). If L is finite-dimensional, then $0 < \kappa(L) < \infty$.

THEOREM 2 (P is algebraically closed, $p > 0$). L is nilpotent iff $\kappa(L) = 1$.

By Levi theorem and theorem 1 the following problem is really interesting for finite-dimensional modular Lie algebras.

PROBLEM 1. For given Lie algebra L over a field of characteristic $p \geq 0$ to find all nonsplit extensions by irreducible modules.

Problem 1 for sl_2 and for Lie algebras of Cartan type $W(m), H(1,1), W(1,1)$ is solved. In particular, nonsplit extensions of simple restricted Lie algebras of rank one is described. Recall that, Zassenhaus algebra $W_1(m) = \langle e_i = x^{(i)} \partial \mid \partial(x^{(j)}) = x^{(j-1)}, -1 \leq i, j \leq p^m - 2 \rangle$ is defined as a Lie algebra of special derivations of divided power algebra. It has one nontrivial central extension

$$V = \langle e_i, z \mid \{e_i, e_j\} = [e_i, e_j] + \delta_{i+j, p^m} (-1)^i z \rangle \text{ (modular Virasoro algebra)}$$

Let $U(L)$ be universal enveloping algebra of Lie algebra L and $\hat{U}(L)$ be its localization over $\langle z \rangle$. Let $\mathcal{L} = \langle e_i \mid 2 \leq i \rangle$ be a nilpotent subalgebra of V .

PROBLEM 2. To study irreducible representations of nonsplit extensions.

THEOREM 3. $\hat{U}(V) \cong U(sl_2) \otimes \hat{U}(\mathcal{L})$.

Problem 2 for other noncentral extensions of $W_1(m)$ is solved. It seems to be hopeful analogies between theories of modular and infinite-dimensional algebras of zero characteristic. For example, Lie algebra of formal differential operators on circle has not only central extension (it is well known as a Virasoro algebra), but also four other irreducible modules with nonsplit extensions.