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ABSTRACT: **Nonabelian K-theory and the K-theory of Kac-Moody Groups**

Nonabelian K-theory is the K-theory of group actions. We construct the K-functors $K_1, K_2, K_3, \ldots$ of this theory, give a simple concrete interpretation of K_1 and K_2 and state a fundamental exact sequence. In general, K_1 takes its values in the category of pointed sets, K_2 in the category of groups, and $K_{i \geq 3}$ in the category of abelian groups. All algebraic K-theories are special cases of the above and groups such as Witt groups and surgery groups, which are normally defined as quotients of K-groups, can be obtained directly above as K-groups of suitable group actions.

The K-theory of Kac-Moody groups is nonabelian K-theory applied to Kac-Moody groups. Browsing through the literature (e.g. J. Tits's Bourbaki Seminar of November 1988), one realizes that the term Kac-Moody group does not refer to a concept, but instead to an ad hoc collection of groups, each of which is called a Kac-Moody group. At the same time (see Tits' Bourbaki Seminar), there is much activity to find such a notion. We propose a concept which we feel is the 'right' one for the following reasons: It is simple, functorial, yields groups defined over all commutative rings containing a fixed ground ring, reduces to the right groups (e.g. GL_n, Sp_{2n}, U_{2n}, etc.) in the classical situation, and the K-theory exact sequences one gets are analogous to those in the K-theory of classical groups. Moreover, it turns out that the group K_2 of a Kac-Moody group over a field is one which is considerably discussed in Tits' Bourbaki Seminar, of course in a non-K-theoretic guise, and Tits' uniqueness theorems for Kac-Moody groups over fields can be extended using K-theoretic techniques to arbitrary commutative rings. One can begin handling questions concerning normal subgroups of Kac-Moody groups over commutative rings and expects solutions, at least in the affine cases, which are similar to those obtained two decades ago by H. Bass for GL_n and the author for all classical groups.