

About the Black Hole Information Paradox:

A Review of S. W. Hawking, *Information Loss in Black Holes*, hep-th/0507171

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Abstract

The information paradox of black holes was raised in 1975 by S. W. Hawking. His argument was based on the fact that particle emission from a black hole, so called the Hawking radiation, cannot convey any information from its interior region. This problem has no definite solution even now.

This year Hawking submitted a paper in which he investigates this problem of information loss in black hole from the view point of quantum gravity using Euclidean path integrals in asymptotically anti-de Sitter (AdS) spacetimes. As a result, it is shown that the path integration over topologically trivial metrics are unitary and information preserving. It implies that quantum gravity preserves information.

I will review this paper introducing some key ingredients of his argument.

1 Introduction

The question of information loss in the black holes was raised by Stephen W. Hawking in 1975 [1]. He claimed that the information locked inside the black hole is lost from this universe after the evaporation of the black hole. His argument was based on the fact that the quantum emission from a black hole, the Hawking radiation [2] is completely random and uncorrelated. This information loss phenomenon in general relativity violates the principle of quantum mechanics in which the time evolution is a reversible process. So this problem was called the information paradox, which shows a serious conflicts between the two theories of physics.

However, from the view point of the the AdS/CFT correspondence [3, 4], the string theory in the AdS₅ spacetime should be unitary because the conformal field theory on the boundary of the AdS spacetime is manifestly unitary. Then no information should be lost.

This year Hawking submitted a paper [5] which shows that the theory of quantum gravity should be also unitary and no information will be lost. I will review this paper introducing some key ingredients of his argument.

2 The Hawking Radiation and the Information Loss

In 1975 Hawking showed that a black hole creates and emits particles as if it is a hot body of temperature $\frac{\hbar\kappa}{2\pi G} \approx 10^{-6} \left(\frac{M_\odot}{M}\right)^\circ\text{K}$, where κ is the surface gravity of the black hole and M_\odot is the solar mass [2]. Because this particle radiation is completely thermal and determined only by the mass M , the angular momentum J and the charge Q of the black hole. So this radiation cannot convey any information from the black hole. This radiation is called the Hawking radiation.

Due to this radiation a black hole lose its mass and eventually evaporates. Here the information of the matter which composed of the black hole is lost from this universe because this information does not come out through the Hawking radiation.

However, this Hawking's argument is based on the quantum field theory on a fixed background spacetime of a black hole, which is only an approximation. According to the precise treatment by Banks et al. [6], this information loss results in serious violation of fundamental laws for physics such as energy conservation, implying that there may be some errors in Hawking's argument.

This information loss problem has been a controversial issue since its appearance, and there was no definite answer for years.

3 AdS/CFT Correspondence and Unitarity

By Maldacena it was conjectured that the theory of superstring on $\text{AdS}_5 \times S^5$ spacetime is the same as $\mathcal{N} = 4$ U(N) Super-Yang-Mills theory, which is one of a conformal field theory, on the 3+1 dimensional boundary of $\text{AdS}_5 \times S^5$ spacetime [4]. Here $\text{AdS}_5 \times S^5$ spacetime is a certain solution of superstring theory, which is a quantum theory of gravity in ten dimensional spacetime.

This correspondence is shown to be valid in some limiting cases. The conjecture is that this correspondence is generally valid and those two theories are exactly same. It has not been proofed yet.

Here the conformal theory is a kind of quantum field theory, and it generally gives a unitary time evolution of states, i.e. the reversible time evolution. However it is not trivial whether the theory of gravity in $\text{AdS}_5 \times S^5$ spacetime is also unitary or not, but this correspondence suggests that it is unitary. Assuming it to be valid, Hawking argues on the information loss in black holes.

4 Euclidean Quantum Gravity

4.1 Procedure

A complete quantum theory of gravity has not been emerged. There are several approaches such as covariant perturbation method and the canonical quantization method, but no one is successful by now.

One of such approaches is the Euclidean quantum gravity [7, 8]. Hawking thinks that this approach is the only sane way to do quantum gravity, and then he applies it to black hole formation and evaporation phenomenon in this research. In this theory the probability amplitude of a state

$$\langle g_2, \phi_2, S_2 | g_1, \phi_1, S_1 \rangle \quad (1)$$

to go from a state with a metric g_1 and matter fields ϕ_1 on the initial surface S_1 to a state with a metric g_2 and matter fields ϕ_2 on the final surface S_2 , is obtained as a sum over all field configurations g and ϕ which take the given values on the surfaces S_1 and S_2 :

$$\langle g_2, \phi_2, S_2 | g_1, \phi_1, S_1 \rangle = \int D[g, \phi] \exp(iI[g, \phi]), \quad (2)$$

where $D[G, \phi]$ is a measure on the space of all field configurations g and ϕ . $I[g, \phi]$ is the action of the fields:

$$I = \frac{1}{16\pi G} \int R\sqrt{-g} d^4x + \frac{1}{8\pi G} \oint K\sqrt{\pm h} d^3x + \int L_m\sqrt{-g} d^4x, \quad (3)$$

where R is the scalar curvature of the spacetime, g is the determinant of the metric. The second integral is done over the boundary surface of the spacetime. K is the extrinsic curvature of the boundary, and h is the induced metric on the boundary. L_m is the Lagrangian of the matter fields. Units are taken to be $c = \hbar = k = 1$.

This action I is real and so the path integral will oscillate and will not converge. One can solve this difficulty by analytically continuing this function to the imaginary time coordinate: rotate the time axis 90° clockwise in the complex plane and replace t by $-i\tau$. This introduces a factor of i into the volume integral of I . Thus the path integral becomes

$$\int D[g, \phi] \exp(-\hat{I}[g, \phi]) \quad (4)$$

where $\hat{I} = -iI$ is the the Euclidean action. The integral with this action is exponentially damped and should therefore converge. The physical amplitude is obtained by rotating back the axis of τ to the Lorentzian time axis of t .

As the boundary condition of this integral, one can take a periodic boundary condition by identifying surfaces of constant time which is separated by an imaginary time interval β . By this procedure the path integral gives the partition function for gravity at temperature β^{-1} , where β is the Euclidean distance between the initial and final surface:

$$Z(\beta) = \int D[g, \phi] \exp(-\hat{I}[g, \phi]) = \text{Tr}(e^{-\beta H}), \quad (5)$$

where H is the Hamiltonian of the system.

However, for an asymptotically flat space this partition function is infinite because the volume of 3-space is infinite. This problem can be solved by adding a small negative cosmological constant Λ to the integrand. It will change the infinity of the spacetime into AdS spacetime. The effective volume of the 3-space reduces to the order of $\Lambda^{-3/2}$ and thus the partition function becomes finite. This change does not affect the formation and evaporation of black holes which is smaller than that 3-space.

4.2 Application to Black Hole Formation and Evaporation

From now we think about the black hole formation and evaporation in this AdS spacetime. In quantum gravity the only observable quantities are the values of fields at infinity, because there is quantum uncertainty in the position of an observation if the observer is in the middle region of the spacetime where the fields are not sufficiently weak. So we focus on the process of the observation of this event from infinity of the spacetime. In this view point this event is thought of as a scattering process: one sends in particles and radiation from infinity and measures what comes out to the infinity. In this research the information is shown to be preserved in this process.

The probability of observing a specific state is given by the path integral over the all metrics of all topologies that fit inside this boundary. Now the boundary at infinity has topology of $S^1 \times S^2$. The simplest topology that fits inside the boundary is the trivial topology $S^1 \times D^3$ where D^3 is the 3-disk. This topology is of the vacuum spacetime metric. The next simplest topology is the first non-trivial topology $S^2 \times D^2$ of the Schwarzschild AdS metric. In this spacetime only one black hole is there and it exists forever. There are other possible topologies that fit into the boundary and summed in the path integral, but these two are the most important cases.

The trivial topology can be foliated by a family of surfaces of constant time each of which has topology of D^3 . In this case one can apply the path integral method of the ordinary quantum field theory in flat spacetime, which gives a unitary mapping between the initial and final quantum states. Then the time evolution in the spacetime of the trivial topology becomes reversible and the information of the initial state is preserved.

This argument does not apply to the non-trivial topology because the spacetime cannot be foliated by such family of surfaces of constant time in this case. An example for this case is the Schwarzschild spacetime, which can be foliated by the family of surfaces each of which has the topology of D^3 with a hole inside of it. They have the black hole horizon as one of their boundary. In this spacetime the correlation function between two points decay exponentially to zero according to the separation between them. This phenomenon can be understood that some portion of the disturbance at the initial point falls into the black hole and the effect of it does not reach to the final point. It is not proofed yet but It is therefore very plausible for the path integral over this topology gives a correlation functions that decay to zero at late Lorentzian times. The proof for this proposition should be given by explicit calculations.

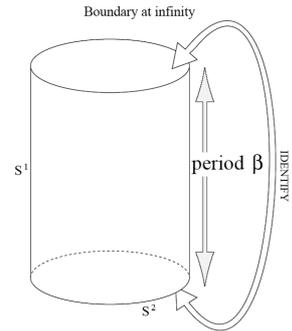


Figure 1: The schematic diagram of the path integral region, asymptotically AdS spacetime. Its boundary has the topology of $S_1 \times S_2$, because the imaginary time coordinate is identified with a period β .

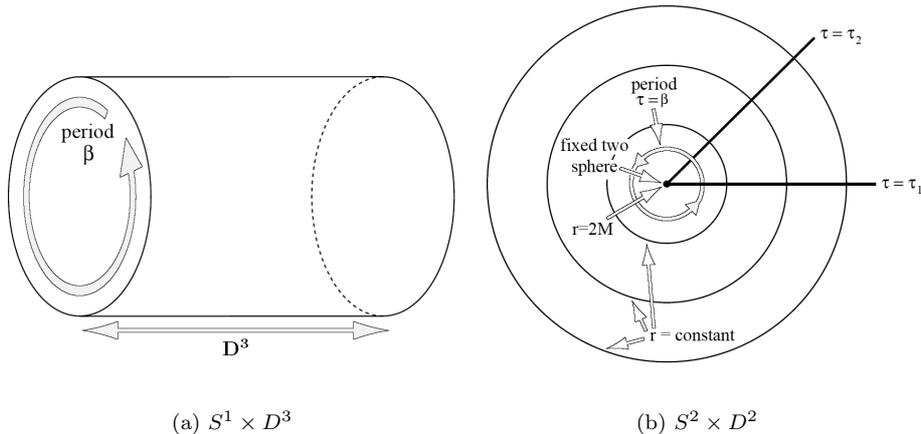


Figure 2: The two simplest spacetime geometry which fit into the boundary of $S^1 \times S^2$. (a) is the topology of vacuum spacetime. (b) is the topology of AdS-Schwarzschild spacetime, which contains an eternal black hole. Radial lines are D^2 , circles and the center point are S^2 .

5 Black Holes in AdS and Information Preservation

For $\beta \ll \lambda$ there are three classical solutions which fit inside the boundary: periodically identified AdS, a small black hole and a giant black hole. Hawking considers each of these black hole solutions.

For giant black holes Maldacena calculated the two point correlation functions in AdS spacetime with an eternal black hole [9]. His calculation showed that the correlation function does not decay to zero in this setting. According to Hawking, this remaining value came from the integration over the metrics of topologically trivial spacetimes, in which the states evolve unitarily. However, in topologically non-trivial spacetimes the states does not evolve unitarily, but the path integration over these metrics gives a correlation function which decays to zero. Based on this fact, Hawking claims that only the unitary path integral will contribute to the correlation function at late time, and thus the information on the initial state will be preserved. In other words, the information of the initial state is preserved in the path integral contribution of topologically trivial metrics, and thus the total process become information preserving, while in the contribution of the topologically non-trivial metrics, like black hole metrics, the information is lost.

Now we move on to the case of small black holes. The giant black holes considered above have very low temperature because they are very massive, then they are stable against the Hawking radiation and will not evaporate away. On the other hand the small black holes, whose mass M satisfy $M \ll \lambda^{-1/2}$, are unstable and eventually evaporate [10]. However, in the setting of now the direct observation of the black hole formation and evaporation is impossible: one can obtain only the correlation function between points at infinity.

According to Hawking, there is no Euclidean geometry which can represent the formation and evaporation of a single black hole, and thus this process should be represented by a superposition of trivial metrics and the metrics of eternal black holes. If this argument is correct, the similar discussion of correlation functions on the boundary applies to this small black hole case. The information is preserved in the contribution from the topologically trivial metrics.

6 Conclusion

Hawking made an argument that quantum gravity is unitary and information of the initial states is preserved in black hole formation and evaporation. Here the only observable quantities are correlation functions between pairs of points at infinity, and they are given by the Euclidean path integrals over metrics of all topologies. In order to define finite and well-defined path integrals,

the integral region is taken to be an asymptotically AdS spacetime.

The integral over topologically trivial metrics can be done by dividing the spacetime into the thin slices of constant time. The integral over each slice will be unitary so the whole path integral will be unitary.

On the other hand the path integral over topologically non-trivial metrics will give a decaying correlation function. In topologically non-trivial spacetimes the path integral will not be unitary, but this effect cannot affect the unitarity of the total process because the contribution to the correlation function from this integral decays to zero.

Then only the information-preserving path integral will contribute to the correlation function, so the total path integral will be unitary.

The proof that the correlation functions decay in topologically non-trivial metrics has not obtained. It is now studied by a Hawking's student, C. Galfard.

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