COMPUTABLE CONDITION FOR THE OCCURRENCE OF NON-UNIFORM HYPERBOLICITY IN FAMILIES OF ONE-DIMENSIONAL MAPS

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In spite of its innoccent definition, the quadratic family

$$f_a(x) = 1 - ax^2$$

can display a wide variety of dynamics which are closely intertwined. Graczyk-Świąteck proved that the set of regular parameters (corresponding to a hyperbolic periodic attractor) is open dense. Lyubich [8] proved that almost every parameter is either regular or stochastic (corresponding to an absolutely continuous invariant measure, *acim* for short). The second possibility in this dichotomy is not negligible, which had earlier been proved by Jakobson [5]. His theorem is recognized as a landmark in the study of chaotic dynamical systems, and so far quite a few different alternative proofs were given [1] [2] [6] [11] [12] [14] [15] [16] [17] [19]. However, any of these arguments tells nothing about the question: *how many parameter values corresponding to acim in the quadratic family?* We give a partial answer to this question. We develop a constructive argument closely following [6], and as a result obtain a lower estimate for the measure of the parameter set corresponding to acim near $a = 2$. Joint work with Stefano Luzzatto.

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