# Zeros of Fourier integrals and the de Bruijn-Newman constant

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For a real constant $\lambda$ let $\Xi_\lambda$ be the entire function defined by

$$\Xi_\lambda(t) = \int_0^\infty z^{5/4} \left(2z\psi''(z) + 3\psi'(z)\right) \exp\left(\frac{\lambda}{4}(\log z)^2 + \frac{it}{2}\log z\right) \frac{dz}{z},$$

where $\psi(z) = \sum_{n=1}^\infty e^{-n^2\pi z}$, so that $\Xi_0$ is Riemann's xi function. For $n = 0, 1, 2, \ldots$ let $\lambda^{(n)}$ be the infimum of the set of real numbers $\lambda$ such that $\Xi_\lambda^{(n)}$ has only real zeros. It is shown that for every $\lambda > 0$ the function $\Xi_\lambda$ has infinitely many zeros, and all but a finite number of them are real and simple. The result implies that the de Bruijn-Newman constant $\Lambda = 4\lambda^{(0)}$ is less than $1/2$, and that $\lim_{n\to\infty} \lambda^{(n)} \leq 0$.

**1 Presented by Author 2**