

The dynamics of elastic planar closed curves under uniform high pressure

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We consider the dynamics of an inextensible elastic closed wire in the plane under uniform high pressure. In 1967, I. Tadjbakhsh and F. Odeh posed a variational problem to determine the shape of a buckled elastic ring under uniform pressure. In order to comprehend a dynamics of the wire, we consider the following two mathematical questions: (i) Can we construct a gradient flow for the Tadjbakhsh-Odeh functional under the inextensibility condition?; (ii) What is a behavior of the wire governed by the gradient flow near every critical points of the Tadjbakhsh-Odeh variational problem? For (i), first we derive a system of equations which governs the gradient flow, and then, give an affirmative answer to (i) by solving this system involving fourth order parabolic equations (Theorem 3). For (ii), we first prove the stability and instability of critical points by considering the second variation formula of the Tadjbakhsh-Odeh functional. Moreover we give a lower bound of its index (Theorem 2). Next we study about a behavior of a solution of the system with an initial data which is close to a critical point of the variational problem (see Theorem 4).