

Title: Class number problems for real and imaginary quadratic fields

Abstract: For imaginary quadratic fields, Gauss conjectured that only nine of them are of class number 1. In '70's, Heegner, Baker and Stark proved this conjecture using modular forms and complex multiplication. Later, Goldfeld-Gross-Zagier gave an effective lower bound for $h(D)$. This yields an effective algorithm to find all imaginary quadratic fields for any given class number.

Meanwhile, for real quadratic fields, we don't have any unified machinery in general as in the imaginary case. We will concentrate on a certain type of real quadratic fields, so called, Richaud-Degert type, whose regulators are of order of $\log D$. We, firstly, review the development in the class number problems, like the Gauss' conjectures in the imaginary case, for Richaud-Degert type of real quadratic fields. Then we explain our results: when the discriminant is not equal to $5 \pmod{8}$, we find the precise upper bound of the discriminant of $h(D) = 1$. This answers to some conjectures in this direction.