

The geography of simply connected 4-manifolds with $b_2^+ = 1$

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One of the fundamental problems in 4-manifolds theory is to find a new family of simply connected smooth (symplectic, complex) 4-manifolds. Though many interesting 4-manifolds have been constructed using techniques such as fiber sum, rational blow-down, knot surgery and so on, it is still very hard to find a new family of 4-manifolds with small Euler characteristic.

Since I discovered a new simply connected symplectic 4-manifold with $b_2^+ = 1$ and $K^2 = 2$ in 2004 by using a rational blow-down surgery, many new simply connected 4-manifolds with small Euler characteristic have been constructed and now it is one of most active research areas in 4-manifolds to find a new family of smooth (symplectic, complex) 4-manifolds with $b_2^+ = 1$.

The aim of this talk is to review recent development in this area. In particular, I'd like to survey the existence and the uniqueness problems of simply connected 4-manifolds with $b_2^+ = 1$ in three levels - smooth category, symplectic category and complex category.