Saturated actions by finite dimensional Hopf $*$-algebras

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If a finite group action α on a unital C^*-algebra M is saturated, the canonical conditional expectation $E : M \to M^\alpha$ onto the fixed point algebra is known to be of index finite type with $Index(E) = |G|$ in the sense of Watatani. More generally if a finite dimensional Hopf $*$-algebra A acts on M and the action is saturated, the same is true with $Index(E) = \dim(A)$. In this paper we prove that the converse is true. Especially in case M is a commutative C^*-algebra $C(X)$ and α is a finite group action, we give an equivalent condition in order that the expectation $E : C(X) \to C(X)^\alpha$ is of index finite type, from which we obtain that α is saturated if and only if G acts freely on X. Actions by compact groups are also considered to show that the gauge action γ on a graph C^*-algebra $C^*(E)$ associated with a locally finite directed graph E is saturated.

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