The Hyperinvariant Subspace Problem for Quasinilpotent Operators

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Let H be a separable, infinite dimensional, complex Hilbert space and $\mathcal{L}(\mathcal{H})$ is the algebra of all bounded linear operators on H. The commutant of T, denoted by $\{T\}'$, is the algebra of all operators X in $\mathcal{L}(\mathcal{H})$ such that $XT = TX$. A closed subspace $M \subset H$ is called a nontrivial hyperinvariant subspace for T if $\{0\} \neq M \neq H$ and $XM \subseteq M$ for each $X \in \{T\}'$. The hyperinvariant subspace problem is the question whether every operator in $\mathcal{L}(\mathcal{H})$ has a nontrivial hyperinvariant subspace. This problem remains still open, especially for quasinilpotent operators in $\mathcal{L}(\mathcal{H})$. In this paper we explore the hyperinvariant subspace problem for quasinilpotent operators. Our main results are related to the stability of extremal vectors for quasinilpotent operators.