1. Research report by Hajime Moriya

1. Summary of my research activity and projects (not limited to the academic year of 2005.) :

My research field is mathematical physics. I have studied some problems that seem relevant to fundamental structures of quantum theory and statistical mechanics. Taking an axiomatic standpoint I tried to provide those with solid mathematical bases by means of operator algebraic methods.

The following is a list of projects that I have worked on.

1. Quantum statistical mechanics. Kubo-Martin-Schwinger (KMS) condition gives a rigorous characterization of equilibrium states for quantum systems (with infinite degrees of freedom). Since its tight connection to operator algebra (Tomita-Takesaki theory) was discovered, it has been an important notion both in pure mathematics and mathematical physics, and a powerful tool in wide range of fields.

KMS condition is given in terms of dynamics. On the other hand, another more intuitive formalism of temperature states is due to the variational principle. I showed the equivalence of those two formalisms discarding previously imposed strict and artificial conditions on quantum dynamics.

2. Von Neumann entropy. Von Neumann entropy is a quantum generalization of Gibbs entropy. I have explored its various basic properties related to quantum statistical mechanics and also to quantum probability.

3. Quantum version of Dynamical entropy. Dynamical entropy gives an invariance for C* or W* dynamical systems. For the classical (commutative) case, thermodynamical entropy (mean entropy density) is a special case of Kolmogorov-Sinai entropy. On the other hand, for the quantum case, such a simple relationship has not yet been shown. A. Connes (noted in his book of Noncommmutative geometry) proposed the following question: Can CNT entropy, a well known formula of quantum dynamical entropy by Connnes Narnhofer and Thirring, provide a variational characterization of equilibrium states? I solved this problem for quantum spin lattice systems and Fermion lattice systems. This result on UHF systems has been now generalized to other C*-systems by several mathematicians.

4. Spontaneous symmetry breaking (SUSY). It is well known that unbroken symmetry is formulated by the following equivalent conditions for quantum spin lattice systems: (1) cluster property of correlation functions (2) factor property of GNS representations (3) triviality of thermodynamical observables. However, when fermions are present, this equivalence generically does not hold. I introduced an appropriate characterization of SUSY based on the quasi-local structure of C*-systems and discussed the univalence superselection rule.

Specific research activities in during the academic year of 2005. :

I have written several research papers (Comm. Math. Phys. in press, J.Math.Phys. published in 2005 March, JMP in press and J.Phys.A. in press.) I participated in two international conferences as an invited speaker. (I submitted the proceedings for them.) I gave a poster presentation at the 2nd Hokkaido COE wakate conference.

I have studied graded (Fermion-Boson) systems. The main result is a rigorous proof of the univalence superselection rule, unbroken symmetry of fermion grading. The univalence superselection rule is considered to be a postulate that nature always fulfills. But we note that there is a known mathematical example violating it. In addition, there are subtle points when transforming quantum spin models to the corresponding fermion models vice versa by Jordan-Wigner transformation. For example, we see that fermion grading symmetry for fermion lattice systems corresponds to 2pi-rotation symmetry in the z-axis for quantum spin lattice systems. While the latter symmetry can be broken, e.g. for XY-models at zero temperature, the former should not be broken in any model according to my result.

I considered properties of quantum correlation in graded systems. The similarity and difference of Markovness for CAR and tensor product systems is clarified.

2. List of published papers

1. H.Araki and H.Moriya
"Equilibrium statistical mechanics of Fermion lattice systems."
Reviews in Mathematical Physics 15(2003) 93-198.

2. H.Araki and H.Moriya
"Local thermodynamical stability of Fermion lattice systems."
Letters in Mathematical Physics 62(2002) 33-45.

3. H.Moriya
"Variational principle and the dynamical entropy of space translation." Reviews in Mathematical Physics 11(1999)1315-1328.

4. H.Moriya
"Entropy density of one-dimensional quantum lattice systems."
Reviews in Mathematical Physics 9(1997)361-369.

5. H.Moriya and Aernout van Enter
"On thermodynamic limits of entropy densities."
Letters in Mathematical Physics 45(1998)323-330.

6. H.Moriya
"Some aspects of quantum entanglement for CAR systems."
Letters in Mathematical Physics 60(2002)109-121.

7. H.Moriya
"On a state having pure-state restrictions for a pair of regions."
Interdisciplinary Information Sciences 10 (2004)31-40.

8. H.Araki and H.Moriya
"Joint extension of states of subsystems for a CAR system."
Communications in Mathematical Physics 237(2003)105-122.

9. H.Moriya
"Validity and failure of some entropy inequalities for CAR systems."
Journal of Mathematical Physics 46 033508 (2005).

10. H.Moriya
"On fermion grading symmetry for quasi-local systems." to appear
Communications in Mathematical Physics.

11. H.Moriya
"Markov property and strong additivity of von Neumann entropy

for graded quantum systems." to appear Journal of Mathematical Physics.

12. H.Moriya
" On separable states for composite systems of distinguishable fermions." to appear Journal of Physics A: Mathematical and General.

3. List of major presentations

Invited talks:

- "Quantum statistical mechanics using $\mathbf{C}^*$-algebraic methods"
  Nihon-Sugakukai, Tokubetsu-Koen (functional analysis), Tokyo, 2003. March.
- "On the univalence superselection rule."
  Quantum probability and infinite dimensional analysis, Levico Italy, 2005.Feb. (Proceeding.)
- "Statistical mechanics for quasi-local graded c* systems."
  4th International Symposium "Quantum Theory and Symmetries" (QTS-4) and 6th International Workshop "Lie Theory and Its Applications in Physics" (LT-6). Workshop: Lie Theory and Its Applications in Physics, Varna Bulugaria, 2005. Aug.

Others

- "Local thermodynamical stability conditions. "
  University of Bourgone 2003.Oct.
- "Entropy inequalities for CAR systems."
  Von Neumann Conference. Budapest Hungary. 2003.Oct.
- "On the univalence superselection rule and characterization of spontaneous symmetry breaking." Hokudai 2nd sugaku sougou COE wakate kenkyukai. 2006.Feb.