Research Report

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The subject that I am interested in is nondestructive testing (NDT). NDT is the method which is used for identifying the internal state of some physical object without destruction. There are several methods for NDT. They are for example, Radiographic Testing, Magnetic Tomography, Electromagnetic Tomography, Ultrasound Tomography and Thermal Tomography.

There are many cases, e.g., in medical or mining probe, that we have to know the internal information without destruction. So the identifying internal inhomogeneities from boundary measurements is very important and interesting problem and there are lots of works related to it. Pólya and Szegö first introduced the Pólya-Szegö tensor which is now called electric polarization tensor (PT) in their works [4, 5]. Studying the PT leads us to know information about the inclusion, e.g., the size of the inclusion. The PT appears in the representation of the perturbed electric potential due to the inclusion with small relative volume. D. J. Cedio-Fenya, S. Moskow and M. Vogelius [2] got an asymptotic formula for the voltage potential in which the PT is contained and they proved the symmetry and positive-definiteness of the first order PT. H. Ammari and H. Kang [1] introduced the notion of the generalized PT and derived the asymptotic expansion with high-order term using the layer potential techniques. By the way, their inclusions are isotropic conductor, that is, their conductivities are real-valued functions. For the case that the inclusion and the back-ground conductor are anisotropic (that is, their conductivities are matrix-valued functions), H. Kang, E. Kim and K. Kim [6] derived the asymptotic polarization tensor (APT). In that paper, they proved the symmetry and positive-definiteness of the first order APT. In [7], we derived in a mathematically rigorous way an asymptotic formula for the effective property, in the context of electrical conductivity, of the medium consisting of inclusions of one material of known shape embedded homogeneously into a continuous matrix of another having material property different from that of the inclusion. One of the significant features of this work is that the shape of the inclusion may be arbitrary and conductivities of the inclusion and the matrix phase are anisotropic. In [9], professor Hyeonbae Kang and I presented a systematic way of computing the polarization tensors, anisotropic as well as isotropic, based on the boundary integral method. We then used this method to compute the anisotropic polarization tensor for ellipses and ellipsoids. The computation reveals the pair of anisotropy and ellipses which produce the same polarization tensors. In [8], we dealt with thermal imaging which is a technique of wide utility in nondestructive testing and evaluation. In that paper, we reconstructed a collection of small inclusions inside a homogeneous object by applying a heat flux and measuring the induced temperature on its boundary. Taking advantage of the smallness of the inclusions, we designed efficient non-iterative algorithms for locating the inclusions from boundary measurements of the temperature. We illustrate the feasibility and the viability of our algorithms by numerical examples.

My present subject of the study in Japan is the mathematical analysis of Thermal Tomography. Mathematically, Thermal Tomography is formulated as follows. First, we apply heat flux on the surface of object which we want to know its internal state. Next we evaluate the resulting distribution of heat on the surface according time. Then, the mathematical analysis of Thermal Tomography is to recover the heat conductivity of an unknown buried inclusion from the resulting heat distribution on the surface.

Recently, Y. Daido, H. Kang and G. Nakamura [3] gave a reconstruction procedure to identify the location of the unknown inclusion for one space dimensional case. The reconstruction procedure given by them is an analogous of the probe method which was introduced by Ikehata to identify the shape of unknown inclusion for the stationary heat conductivity case. They introduced a new indicator function which was used to indicate the location of the unknown inclusion.

Now, a very natural question is to ask the possibility of extending their result to higher space dimension. Professor Gen Nakamura and I have been working on this project since I came here Hokkaido University in 2006. The difficulty of extending their result lies in how to analyze the so called reflected solution. We remark that a sum of the reflected solution and the fundamental solution of the operator for the case without inclusion gives the fundamental solution of the operator for the case there is a inclusion. Under some mathematically idealized situation, we provide a mathematically rigorous scheme to reconstruct the unknown inclusion by boundary measurements. This work is conducted with professor Gen Nakamura and professor Victor Isakov [10]. This result is proved in $\mathbb{R}^3$. Based on this work, professor Gen Nakamura, Lei Yi and I show that the same result can be hold in $\mathbb{R}^2$ and also show numerical computation [11].

References

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List of Publications

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[7] Polarization tensors and effective properties of anisotropic composite materials (with H.Ammari and H.Kang), *J. Differential Equations* 215 (2005) 401-428.

[8] A direct algorithm for thermal imaging of small inclusions (with H.Ammari, E.Iakovleva, and H.Kang), *SIAM J. Multiscale Modeling and Simulation* 4 (2005), 1116-1136.

[9] Anisotropic polarization tensors for ellipses and ellipsoids (with H.Kang), *Jour. Comp. Math.* 25 (2) (2007), 157-168.

[10] V. Isakov, K. Kim and G. Nakamura, Reconstruction of an unknown inclusion by thermography, submitted.

[11] Lei Yi, K. Kim and G. Nakamura, Numerical implementation for a 2-D thermal inhomogeneity through the dynamical probe method, submitted.

List of Presentations

1. 2005. 10. 4 : SNU-HU Joint Symposium, Seoul National University, Korea.
2. 2007. 12. 7 - 11 : Fourth Pacific Rim Conference, City University of Hong Kong, Hong Kong.
3. 2008. 2. 21 - 22 : Northeastern Symposium On Mathematical Analysis, Hokkaido University, Japan