Post-Doctoral Research Report
(From June 2007)

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RESEARCH REPORT

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1. Dynamics of a discrete Turing model of morphogenesis. I have studied a discrete version of the Turing continuous model of morphogenesis. This model is described by a partial differential equation of evolution type. When discretized, it constitutes a lattice dynamical system, called a coupled map lattice.

So far, I have found many interesting dynamical and hyperbolic properties from this system. Especially, I found a spatio-temporal chaos of this lattice model. In 1999, firstly, a spatio-temporal chaos was obtained from the lattice system of the Maginu model of morphogenesis which is simplified from the Turing model of morphogenesis.

Besides, the local map of this lattice dynamical system has intriguing asymptotic behaviors. More precisely, qualitative and quantitative dynamical properties on all the trajectories escaping to infinity and all those remained in a bounded domain has been investigated, such as invariant measures, Julia sets, deviations, and so forth.

2. Spatio-Temporal Intermittency. In 1995, J. Bricmont and A. Kupiainen initiated the study of statistical properties of a special type of CML, a lattice of weakly coupled expanding circle maps, but they did not use any thermodynamics approaches, such as entropies and variational principles. In their work, an absolutely continuous invariant measure displaying spatio-temporal chaos was constructed by using the Perron-Frobenius operator. On the other hand, in 1996, M. Jiang and Y. Pesin found an invariant measure with mixing property for CMLs with hyperbolic local map by using the thermodynamic formalism of the infinite-dimensional case, but the absolute continuity of the measure was not discussed in their work.

I am now examining whether or not this measure is absolutely continuous, and then, in the case that it is not absolutely continuous or it is difficult to check it, I plan to build a new invariant mixing measure which is absolutely continuous by using the Perron-Frobenius operator or other compatible methods. Additionally, finding an invariant measure for the CMLs with non-hyperbolic local map, or, at least, showing the existence of such a measure is another challenge on my current research.

List of published papers

- *Dynamics of Local Map of a Discrete Brusselator Model: Eventually Trapping Region and Strange Attractors*, DCDS-A, **20:4** (2008), 939–959.

- *Chaos in Traveling Waves in Lattice Systems of Unbounded Media*, Technical Report Series 16th, **117** (2008).

List of major presentations

- *Kyoto Dynamical System Seminar* at Kyoto University on Oct. 12th, 2007. Title: Chaotic Traveling Waves in Coupled Map Lattices of Unbounded Media.

- *Ergodic Theory and its Applications* at Keio University on Dec. 19th, 2007. Title: Spatio-Temporal Chaos in Lattice Dynamical Systems.

- *Dynamical System Meeting at Karuizawa* (Nihon Univ.) on Jan. 8th, 2008. Title: Asymptotic behaviors in a discrete Turing model.

- *Workshop on Dynamical Systems & Related Fields* at Ajou University (Seoul) on Feb. 18th, 2008. Title: Spatio-Temporal Chaos in Lattice Systems of Unbounded Media.