RESEARCH REPORT

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1. **Research Activities**

I have been working on several issues on Partial Differential Equations (PDEs) in 2006. I studied the semirelativistic Hartree equations and (in)compressible Navier-Stokes equations for the fluids with vacuum.

2. **Research background**

1. *Semirelativistic Hartree equations*: The physical models are originated from the Boson stars which are quantum mechanical system of massive or massless Boson particles with Coulomb or gravitational interaction defined by the convolution with $|x|^{-\gamma}$ for some positive number γ less than the space dimension n. The equation is to be derived by the method of mean field dynamics. The main problem concerning this equation is to establish the global well-posedness of solutions and their scattering according to the value of γ.
2. *Navier-Stokes equations*: The global existence of weak solution of Navier-Stokes equation with vacuum is well-known but the uniqueness remains still open even for two dimensional case. It has been known that to guarantee the uniqueness, a strong regularity is inevitable. But there has almost not been known for a general fluid with vacuum. In general, the regularity of solution is mainly gained by the parabolicity of the momentum equation. But in the presence of the vacuum, the momentum equation looses the parabolicity. Thus we need a compatibility condition which turns out to be necessary and sufficient for a solution to have the strong regularity. Here the problem is how to get a high regularity of solution and extend to more complicated models.

3. Research Methods

1. *Semirelativistic Hartree equations*: The main tool is the Strichartz estimate associated with Klein-Gordon equation. We also need a time decay of linear solution for which we use the finite propagation speed of solutions.
2. *Navier-Stokes equations*: We get a uniform bound of linear solutions is necessary to use the usual iteration scheme. We first control the density of solution and then the velocity(temperature and their high regularity) via a bootstrap argument.

4. Results

1. *Semirelativistic Hartree equations*: In the papers [5,12], the global time behavior was shown by a careful observation of the conservation laws and Strichartz estimates. In particular the radially symmetric solutions were considered in [12]. In [13] we considered a potential which is the sum of two potentials with different decay rates. One of the most interesting thing is the behavior of solutions when the mass is large (we call this nonrelativistic limit). In [5] we showed the solution

with large mass of semirelativistic equation behaves like a solution of Schrödinger equation with the same nonlinearity.
2. *Navier-Stokes equations* In [4], we discussed a polytropic compressible fluid with vacuum. We applied the method in [4] to the regularity problem of the barotropic compressible Navier-Stokes equations [11] and heat conducting incompressible Navier-Stokes equations [9]. The most important results is the local strong solvability which is sharp in view of the recent result [2] where we proved that there is no global strong solution if the initial density has compact support.

5. **Papers from 2006**

[1] Y. Cho and H. Kim, *On classical solutions of the compressible Navier-Stokes equations with nonnegative initial densities*, Manuscripta Mathematica (1) **120** (2006), 91-120.
[2] Y. Cho and B. J. Jin, *Blow-up of viscous heat-conducting compressible flows*, J. Math. Anal. Appl. (2) **320** (2006), 819-826.
[3] Y. Cho and T. Ozawa, *Remarks on modified improved Boussinesq equations in one space dimension,* Proc. Roy. Soc. A (2071) **462** (2006), 1949-1963.
[4] Y. Cho and H. Kim, *Existence results for viscous polytropic fluids with vacuum,* Journal of Differential Equations **228** (2006), 377-411.
[5] Y. Cho and T. Ozawa, *On the semi-relativistic Hartree type equation*, SIAM J. Math. Anal. **38** (2006), 1060-1074.
[6] Y. Cho and T. Ozawa, *On small amplitude solutions to the generalized Boussinesq equations*, Discrete and Continuous Dynamical Systems **17** (2007), 691-711.
[7] Y. Cho, S. Lee and Y. Shim, *A maximal inequality associated to Schrödinger type equation,* Hokkaido Math. J. **35** (2006), 767-778.
[8] Y. Cho and T. Ozawa, *Global existence on nonlinear Schrödinger-IMBq equations,* J. Math. Kyoto Univ. **46** (2006), 535-552.
[9] Y. Cho and H. Kim, *Existence result for heat-conducting viscous incompressible fluids with vacuum,* to appear in J. Korean Math. Soc.; Hokkaido Univ. Preprint Series in Math. ♯742 (2005).
[10] Y. Cho and Y. Shim, *Global estimates of maximal operators generated by dispersive equations,* Hokkaido Univ. Preprint Series in Math. ♯704 (2005).
[11] Y. Cho, *High regularity of solutions of compressible Navier-Stokes equations*, to appear in Advances in Differential Equations; Hokkaido Univ. Preprint Series in Math. ♯776 (2006).
[12] Y. Cho and T. Ozawa, *On radial solutions of semi-relativistic Hartree equations*, to appear in DCDS Supplement 2007; Hokkaido Univ. Preprint Series in Math. ♯792 (2006).
[13] Y. Cho and T. Ozawa, *Global solutions to semirelativistic Hartree equations*, Hokkaido Univ. Preprint Series in Math. ♯813 (2006).
[14] Y. Cho, T. Ozawa, H. Sasaki and Y. Shim, *Remarks on the relativistic Hartree equations* Hokkaido Univ. Preprint Series in Math. ♯827 (2007).
[15] Y. Cho, T. Ozawa and Y. Shim, *Elliptic estimates independent of domain expansion* in preparation.

6. **Talks**

[1] *Unique solvability of the initial boundary value problems for compressible viscous fluids,* International Conference on Nonlinear PDE and Related Topics: Celebrating Neil Trudinger's 60th Birthday, Australian National University, Australia, 2002.07
[2] *Local existence for viscous polytropic fluid with vacuum,* The 1st PDE Workshop of Educational Science Institute, Cheju National University, Korea, 2003.04
[3] *Local existence for heat-conducting incompressible fluid,* The 6th Workshop on Differential Equations, Mathematical Research Center, Chonnam National University, Korea, 2003.08
[4] *Regularity results for viscous compressible fluids with vacuum,* The 12th Applied Mathematical Forum, SoAnbo, Korea, 2004.02
[5]*Blow-up of the viscous heat-conducting compressible flow,* The 6th Northeastern Symposium on Mathematical Analysis, Tohoku University, Japan, 2005.02
[6] *Sharp boundedness of Bochner-Riesz operator with negative index,* PDE Seminar, Hokkaido University, Japan, 2005.03
[7] *Lorentz space extension of Strichartz estimates,* Wave Seminar, Hokkaido University, Japan, 2005.04
[8] *On the Bochner-Riesz operator with negative index,* Harmonic Analysis and Partial Differential Equation, University of Kiel, Germany, 2005.06
[9] *On classical solutions of the compressible Navier-Stokes equation with nonnegative density,* The 15th PDE Real Analysis Seminar, University of Tokyo, Japan, 2005.07
[10] *An improvement of Bochner-Riesz problem,* Math Seminar, KIAS, Korea, 2005.07
[11] *Boundedness of Fourier multiplier operator defined by elliptic type function,* Harmonic Analysis Symposium "Harmonic Analysis and its Application at Sapporo," Hokkaido University, Japan, 2005.08
[12] *Long time behavior of small amplitude solutions of generalized Boussinesq and modified improved Boussinesq equations*, Sapporo Guest House Symposium 20 "Nonlinear Wave Equations," Sapporo Guest House, Japan, 2005.11
[13] *Well-posedness of semi-relativistic Hartree type equation*, Wave Seminar, Hokkaido University, Japan, 2006. 05
[14] *On some elliptic estimates*, Sapporo Guest House Symposium 22 "Nonlinear Wave Equations," Sapporo Guest House, Japan, 2006.11