

# Arithmetic and Algebraic Geometry 2014

Organizers: T. Katsura (Hosei Univ.)  
I. Nakamura (Hokkaido Univ.)  
T. Terasoma (Univ. Tokyo)

Dates : January 27 (Monday)–January 30 (Thursday), 2014

Place: Graduate School of Mathematical Sciences

The University of Tokyo

Program:

January 27 (Monday)

10:00–11:00 : Jun-Muk Hwang (KIAS)

Minimal rational curves on Veronese double cones

11:20–12:20 : Keiji Oguiso (Osaka Univ.)

Automorphism groups of compact hyperkaehler manifolds of small Picard numbers

13:40–14:40 : Matthias Schuett (Leibniz Univ. Hannover )

64 Lines on smooth quartic surfaces

15:10–16:10 : Fumiharu Kato (Kumamoto Univ.)

Combinatorial Li-Yau inequality and rational points on curves

16:30–17:30 : Ichiro Shimada (Hiroshima Univ.)

On the supersingular K3 surface in characteristic 5 with Artin invariant 1

January 28 (Tuesday)

10:00–11:00 : JongHae Keum (KIAS)

Automorphisms of large order of K3 surfaces

11:20–12:20 : Olivier Debarre (ENS)

Quadratic line complexes

13:40–14:40 : Tohru Eguchi (Rikkyo Univ.)

Mathieu Moonshine and Superconformal Symmetry

15:10–16:10 : Kazushi Ueda (Osaka Univ.)

Mirror symmetry and K3 surfaces

16:30–17:30 : Tetsuji Shioda (Rikkyo Univ.)

Weierstrass transformation and cubic surfaces

18:00–19:45: Banquet

January 29 (Wednesday)

10:00–11:00 : Takeshi Saito (Univ. Tokyo)

Characteristic cycle of a constructible sheaf on a surface

- 11:20–12:20 : Yongnam Lee (KAIST)  
Q-Gorenstein deformation and its applications
- 13:40–14:40 : Kazuhiro Fujiwara (Nagoya Univ.)  
Rigid geometric interpretations of cohomology theories
- 15:10–16:10 : Naoki Imai (Univ. Tokyo)  
Good reduction of affinoids for epipelagic representations in the Lubin-Tate perfectoid space
- 16:30–17:30 : Yoichi Mieda (Kyoto Univ.)  
On irreducible components of Rapoport-Zink spaces
- January 30 (Thursday)
- 10:00–11:00 : Lin Weng (Kyushu Univ.)  
Motivic Euler Product and its Applications
- 11:20–12:20 : Atsushi Shiho (Univ. Tokyo)  
On differential Artin conductor of overconvergent isocrystals
- 13:40–14:40 : Kei Hagihara (Hokkaido Univ.)  
Riemann-Roch theorem for logarithmic schemes
- 15:00–16:00 : Takao Yamazaki (Tohoku Univ.)  
A vanishing result for a K-group of reciprocity functors

# Arithmetic and Algebraic Geometry 2014

科学研究費補助金基盤研究 (S)No.23224001 (代表者：中村郁), 東京大学大学院  
数理科学研究科・卓越した大学院拠点形成支援補助金「数学新展開の研究教育拠点」  
(リーダー：川又雄二郎) により、標記のような研究集会を下記の要領で開催します  
のでご案内致します。

Organizers: 桂 利行 (法政大)  
寺杣 友秀 (東京大)  
中村 郁 (北海道大)

## 記

日時：2014年1月27日(月) - 1月30日(木)

場所：東京大学大学院数理科学研究科大講義室

(京王井の頭線駒場東大駅下車徒歩5分)

プログラム：

1月27日(月)

10:00-11:00 : Jun-Muk Hwang (KIAS)

Minimal rational curves on Veronese double cones

11:20-12:20 : 小木曾 啓示 (大阪大)

Automorphism groups of compact hyperkaehler manifolds of small Picard numbers

13:40-14:40 : Matthias Schuett (Hannover Univ.)

64 Lines on smooth quartic surfaces

15:10-16:10 : 加藤 文元 (熊本大)

Combinatorial Li-Yau inequality and rational points on curves

16:30-17:30 : 島田 伊知朗 (広島大)

On the supersingular K3 surface in characteristic 5 with Artin invariant 1

1月28日(火)

10:00-11:00 : JongHae Keum (KIAS)

Automorphisms of large order of K3 surfaces

11:20-12:20 : Olivier Debarre (ENS)

Quadratic line complexes

13:40-14:40 : 江口 徹 (立教大)

Mathieu Moonshine and Superconformal Symmetry

15:10-16:10 : 植田 一石 (大阪大)

Mirror symmetry and K3 surfaces

16:30-17:30 : 塩田 徹治 (立教大)

Weierstrass transformation and cubic surfaces

18:00-19:45: レセプション

1月29日(水)

10:00–11:00 : 齋藤 毅 (東京大)

Characteristic cycle of a constructible sheaf on a surface

11:20–12:20 : Yongnam Lee (KAIST)

Q-Gorenstein deformation and its applications

13:40–14:40 : 藤原 一宏 (名古屋大)

Rigid geometric interpretations of cohomology theories

15:10–16:10 : 今井 直毅 (東京大)

Good reduction of affinoids for epipelagic representations in the Lubin-Tate perfectoid space

16:30–17:30 : 三枝 洋一 (京都大)

On irreducible components of Rapoport-Zink spaces

1月30日(木)

10:00–11:00 : Lin Weng (九州大)

Motivic Euler Product and its Applications

11:20–12:20 : 志甫 淳 (東京大)

On differential Artin conductor of overconvergent isocrystals

13:40–14:40 : 萩原 啓 (北海道大)

Riemann-Roch theorem for logarithmic schemes

15:00–16:00 : 山崎 隆雄 (東北大)

A vanishing result for a K-group of reciprocity functors

**ABSTRACTS OF LECTURES  
ARITHMETIC AND ALGEBRAIC GEOMETRY 2014  
JANUARY 27 - 30, UNIVERSITY OF TOKYO**

**Olivier Debarre (École normale supérieure, France)  
Quadratic line complexes.**

**Abstract.** In this talk, a quadratic line complex is the intersection, in its Pluecker embedding, of the Grassmannian of lines in an 4-dimensional projective space with a quadric. We study the moduli space of these Fano 5-folds in relation with that of EPW sextics.

**Tohru Eguchi (Rikkyo University, Japan)  
Mathieu Moonshine and Superconformal Symmetry.**

**Abstract.** A few years ago we have found a new example of a moonshine phenomenon by expanding elliptic genus of K3 surface in terms of characters of N=4 superconformal algebra. This example is called as the Mathieu moonshine since expansion coefficients decompose into a sum of representations of Mathieu group M24. More recently generalizations of Mathieu moonshine have been discovered corresponding to each of the 23 Niemeier lattices. We discuss our present understanding of these moonshine phenomena.

**Kazuhiro Fujiwara (Nagoya University, Japan)  
Homotopy construction by Frobenius dynamics in étale topology.**

**Abstract.** In this lecture, I plan to discuss "homotopy constructions using Frobenius", which are universal to cohomology theories. Some applications involving rigid-étale theory will be given.

**Kei Hagihara (Hokkaido University, Japan)  
Riemann-Roch theorem for logarithmic schemes.**

**Abstract.** The Riemann-Roch theorem is a fundamental tool for the computation of cohomological invariants of algebraic varieties or complex manifolds, and plays an essential role in various fields of mathematics, such as complex analysis, algebraic geometry and number theory.

In this talk, we present a generalisation of the theorem to logarithmic varieties (in the sense of Fontaine-Illusie-Kato), which is formulated by a log-geometric variant of K-groups - a Kummer étale K-group, and give its application to number theory.

**Jun-Muk Hwang (KIAS, Korea)**  
**Minimal rational curves on Veronese double cones**

**Abstract.** For a Fano manifold  $X$  of Picard number 1 and a general point  $x \in X$ , the tangent directions to rational curves of minimal degree through  $x$  form a projective variety in the projectivized tangent space of  $X$  at  $x$ . This projective variety is called the variety of minimal rational tangents (VMRT) at  $x$ . VMRT plays an important role in many questions regarding the geometry of  $X$  and is particularly useful if it is nonsingular, which is true in many examples. For this reason, it has been asked whether VMRT at a general point is always a nonsingular subvariety. In a joint work with Hosung Kim, we give a negative answer to this question: when  $X$  is a double cover of a Veronese cone of dimension  $n$  associated to a general hypersurface of degree  $2d$  where  $d$  is an odd integer bigger than 2, the VMRT at a general point of  $X$  is not smooth if  $2d \leq n$ .

**Naoki Imai (University of Tokyo, Japan)**  
**Good reduction of affinoids for epipelagic representations in the  
 Lubin-Tate perfectoid space**

**Abstract.** We construct a family of affinoids in the Lubin-Tate perfectoid spaces, and study the cohomology of their reductions. Further, we will show that the cohomology realizes the local Langlands correspondence and the local Jacquet-Langlands correspondence for epipelagic representations of  $GL(n)$ . This is a joint work with Takahiro Tsushima.

**Fumiharu Kato (Kumamoto University, Japan)**  
**Combinatorial Li-Yau inequality and rational points on curves**

**Abstract.** (Joint-work with Gunther Cornelissen and Janne Kool) A combinatorial Li-Yau inequality, which is the main theorem of this talk, gives a lower bound of ‘gonality of graphs’ in terms of the first Laplacian eigenvalue and some other graph-theoretic invariants. Through the stable reduction of covering maps between curves, it induces a lower bound of gonality of curves. As the classical Li-Yau inequality provides useful lower bound of gonality of modular curves, useful for example to show several finiteness results on rational points, our combinatorial version gives similar set of results for Drinfeld modular curves in positive characteristics.

**JongHae Keum (KIAS, Korea)**  
**Automorphisms of large order of K3 surfaces**

**Abstract.** Recently, I have determined all possible orders of automorphisms of K3 surfaces in all characteristics except 2 and 3.

In particular, 66 is the maximum possible finite order in each characteristic bigger than 3. Over the complex number field the result is simple: an interger  $N$  is the order of an automorphism of a complex K3 surface iff the Euler functional value of  $N$  is bounded by 20.

A question arises: for a given order  $N$ , what are the possible non- symplectic orders? I will answer to this question for all  $N$  whose Euler functional value is bigger than 8.

For  $N$  not divisible by 7, the non-symplectic order is unique, e.g. for  $N = 66$ , the non-symplectic order is 66, that is, any automorphism of order 66 is purely non-symplectic, and for  $N = 60$ , the non-symplectic order is 12, that is, any automorphism of order 60 has non-symplectic order 12. For  $N$  divisible by 7, i.e.  $N = 42, 28$ , or 21, there are multiple possibilities for the non-symplectic order.

**Yongnam Lee (KAIST, Korea)**

**Q-Gorenstein deformation and its applications**

**Abstract.** In this talk we will discuss Q-Gorenstein schemes and Q-Gorenstein morphisms in a general setting. Based on the notion of Q-Gorenstein morphism, we define the notion of Q-Gorenstein deformation and discuss its properties. Versal property of Q-Gorenstein deformation and its applications are also considered. This is joint work with Noboru Nakayama.

**Yoichi Mieda (Kyoto University, Japan)**

**On irreducible components of Rapoport-Zink spaces.**

**Abstract.** A Rapoport-Zink space is a moduli space of deformations by quasi-isogenies of a  $p$ -divisible group with additional structures. It is a locally noetherian formal scheme which is not quasi-compact in general. In this talk, we give some finiteness result on its irreducible components. We will also explain its application to  $l$ -adic cohomology of the Rapoport-Zink tower, which is expected to be closely related to the local Langlands correspondence.

**Keiji Ogusio (Osaka University, Japan)**

**Automorphism groups of compact hyperkaeler manifolds of small Picard numbers**

**Abstract.** After recalling Tits' alternatives of birational automorphism groups of compact hyperkaehler manifolds, I would like to explain the structure of birational automorphism groups - which coincide with the biregular ones in our case - of particular hyperkaehler fourfolds, the Hilbert schemes of two points of K3 surfaces found by Cayley. They provide explicit examples of hyperkaehler manifolds of the smallest possible Picard number with "essentially non-commutative" automorphism group, in dimension greater than two.

**Takeshi Saito (University of Tokyo, Japan)**

**Characteristic cycles of a constructible sheaf.**

**Abstract.** The analogy between the irregularity of  $D$ -modules and the wild ramification of  $l$ -adic sheaves suggests that the characteristic cycle should be defined also for the latter and should enable to compute the total dimension of the vanishing cycles as an intersection number for an isolated singularity of a morphism to a curve. I will report on recent progresses on this subject in the tamely ramified case and in the 2-dimensional case.

**Matthias Schuett (Leibniz University Hannover, Germany)**  
**64 Lines on smooth quartic surfaces**

**Abstract.** In a 1943 paper, Benjamino Segre claimed that a smooth complex quartic surface contains at most 64 lines. However, his arguments turn out to be incomplete, if not plain wrong at some places. I will present joint work with Slawomir Rams which uses elliptic fibrations to give a complete proof of the corresponding statement over any field of characteristic other than 2 and 3.

**Atsushi Shiho (University of Tokyo, Japan)**  
**On differential Artin conductor of overconvergent isocrystals**

**Abstract.** Differential Artin conductor is a quantity which measures the ramification of an overconvergent isocrystal along boundary and it is defined via the theory of p-adic differential equations on p-adic annuli. In this talk, we prove that the differential Artin conductor of a given overconvergent isocrystal on a smooth variety is equal to that of its restriction to most curves.

**Ichiro Shimada (Hiroshima University, Japan)**  
**On the supersingular K3 surface in characteristic 5 with Artin invariant 1**

**Abstract.** We exhibit three projective models of  $X$  with large automorphism group. Moreover, by using the superspecial abelian surface, we construct an interesting configuration of 96 smooth rational curves on  $X$ . This is a joint work with T. Katsura and S. Kondo.

**Tetsuji Shioda (Rikkyo University, Japan)**  
**Weierstrass transformation and cubic surfaces.**

**Abstract.** We hope to clarify the classical subject of cubic surfaces that they arise from the projective plane  $\mathbf{P}^2$  by blowing up 6 points. Let

$$P(s) = (s/(s-1)^2 : s/(s-1)^3 : 1) \text{ in } \mathbf{P}^2 \text{ for } s \neq 0, 1.$$

**Theorem.** Take the 6 distinct points  $P_i = P(s_i)$  ( $i = 1, \dots, 6$ ). Then the cubic surface  $V$  is defined by the Weierstrass equation

$$y^2 + txy = x^3 + x(p_0 + p_1t + p_2t^2) + q_0 + q_1t + q_2t^2 + t^3,$$

in the homogeneous coordinate  $(x : y : t : 1) = (X : Y : T : Z)$  of  $\mathbf{P}^3$  and the 6 lines  $L_i$  (= blowup of  $P_i$ ) are given by

$$X = aT + bZ, \quad Y = dT + eZ \quad (a, b, d, e : \text{constants})$$

$$a = -s_i r, \quad \text{where } r^3 = s_1 \dots s_6.$$

Further  $V$  is smooth iff the products of any three (or six)  $s_i$  are not equal to 1.

The proof uses the idea of Weierstrass transformations and Mordell-Weil lattices.

**Takao Yamazaki (Tohoku University, Japan)**  
**p-adic soliton theory and arithmetic geometry.**

**Abstract.** p-adic soliton theory was introduced in 1994 by Anderson and was applied to arithmetic problems such as Manin-Mumford conjecture. He estimated the number of p-torsion points on the theta divisor of a quotient of the Fermat curve. After a survey of recent developments in p-adic soliton theory, we explain our result generalizing Anderson's result to more general curves. (Joint work with S. Kobayashi.)

**Lin Weng (Kyushu University, Japan)**  
**Motivic Euler Products and its Applications**

**Abstract.** We begin with a construction of motivic zeta functions for curves over any base field, using moduli stacks of semi-stable bundles. Based on it, we define motivic Euler products. As applications, we formulate the corresponding Tamagawa number conjecture, offer a pair of intrinsic relations between total motivic mass of principal bundles and its semi stable parts, using parabolic reduction. If time is allowed, we will also explain how these works can be used to uniformize the arithmetic approach of Harder-Narasimhan and the geometric approach of Atiyah-Bott on Poincare series for moduli spaces of bundles.