DEGENERATION OF ORLIK-SOLOMON ALGEBRAS AND MILNOR FIBERS OF COMPLEX LINE ARRANGEMENTS

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Let $\bar{\mathcal{A}} = \{\overline{L}_0, \overline{L}_1, \ldots, \overline{L}_n\}$ be an arrangement of $n+1$ lines in the complex projective plane $\mathbb{P}^2_{\mathbb{C}}$, with defining polynomial $Q(x, y, z)$ and Milnor fiber $F := \{(x, y, z) \in \mathbb{C}^3 \mid Q(x, y, z) = 1\}$. We consider the monodromy automorphism on the Milnor Fiber

$$\begin{aligned} h: \quad & F \to F \\ & (x, y, z) \mapsto e^{2\pi i/(n+1)} \cdot (x, y, z), \end{aligned}$$

and the monodromy operator induced at cohomology level $h^1 : H^1(F, \mathbb{C}) \to H^1(F, \mathbb{C})$. The first cohomology group is decomposed into direct sum of eigenspaces as

$$H^1(F, \mathbb{C}) = \bigoplus_{\lambda^{n+1}=1} H^1(F)_\lambda,$$

where the direct sum runs over nonzero complex number $\lambda \in \mathbb{C}^\times$ with $\lambda^{n+1} = 1$. The computation of the eigenspaces $H^1(F)_\lambda$ in terms of the arrangements combinatorics is an open question in hyperplane arrangement theory.

We give a vanishing theorem for these eigenspaces, for projective line arrangements such that there exists a line containing at most one point of multiplicity greater or equal to 3. The result is deduced from the vanishing of the cohomology of certain Aomoto complex over finite fields, and the modular bound of the local system cohomology groups given by Papadima-Suciu.
I will explain the result, give a brief sketch of proof, and speak about two degeneration homomorphisms that could be be very interesting for future conjectures.