Residues, Dynamics and Hyperfunctions

Dates: July 24 – July 28, 2017.
Venue: Room 4-501 in bldg no.4, Faculty of Science, Hokkaido University.
Organizers: Toru Ohmoto (Hokkaido), Naofumi Honda (Hokkaido).

http://hokkaidosummerinstitute.oia.hokudai.ac.jp/courses/CourseDetail=G073
http://www.math.sci.hokudai.ac.jp/en/seminar-index/
workshop-residues-dynamics-and-hyperfunctions-3.php

The program includes a course of summer school, "Complex Analytic Geometry - Residues and Fixed Points", in which two sequential lectures are given by emeritus Prof. Suwa (Hokkaido) and Prof. Abate (Pisa). The workshop is partly supported by Hokkaido University Summer Institute and KAKENHI grant no.17H02838 (T. Ohmoto).

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Mon (July 24)

13:00-14:30 M. Abate (Pisa)
Residues, meromorphic connections and local holomorphic dynamics I
14:45-16:15 T. Suwa (Hokkaido)
Relative de Rham, relative Dolbeault cohomologies and their applications I

Thu (July 25)

08:45-10:15 M. Abate (Pisa)
Residues, meromorphic connections and local holomorphic dynamics II
10:30-12:00 T. Suwa (Hokkaido)
Relative de Rham, relative Dolbeault cohomologies and their applications II

13:30-14:30 J. Raissy (Toulouse)
Automorphisms of $\mathbb{C}^2$ with an invariant Fatou component biholomorphic to $\mathbb{C} \times \mathbb{C}^*$
14:45-15:30 K. Fujisawa (Hokkaido, Grd)
Thom form in equivariant Čech-de Rham theory
16:00-17:00 D. Marti-Pete (Kyoto)
The escaping set of transcendental self-maps of the punctured plane

Wed (July 26)

08:45-10:15 M. Abate (Pisa)
Residues, meromorphic connections and local holomorphic dynamics III
10:30-12:00 T. Suwa (Hokkaido)
Relative de Rham, relative Dolbeault cohomologies and their applications III

Thu (July 27)

08:45-10:15 M. Abate (Pisa)
Residues, meromorphic connections and local holomorphic dynamics IV
10:30-12:00 T. Suwa (Hokkaido)
Relative de Rham, relative Dolbeault cohomologies and their applications IV

13:30-14:30 S. Tajima (Tsukuba)
Local cohomology, Grothendieck local residues and algorithms
14:45-15:30 D. Komori (Hokkaido, Grd)
Intuitive representation of local cohomology groups
16:00-17:00 N. Honda (Hokkaido)
Several operations on Sato hyperfunctions from a viewpoint of Čech-Dolbeault cohomology
18:00- Banquet

Fri (July 28)

09:30-10:30 T. Kawahira (Tokyo Inst. Tech.)
The Riemann hypothesis and holomorphic index in complex dynamics
10:45-11:45 T. Ueda (Kyoto)
Fixed points of some polynomial automorphisms of $\mathbb{C}^n$

13:30-14:30 M. Abate (Pisa)
Dynamics of families of maps tangent to the identity
14:45-15:45 T. Asuke (Tokyo)
Characteristic classes for infinitesimal deformations of foliations
16:00-17:00 T. Suwa (Hokkaido)
Relative Bott-Chern cohomology

Abstracts: mini-courses

Relative de Rham, relative Dolbeault cohomologies and their applications

Tatsuo Suwa (Hokkaido University)
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Here are the key words for each time slot of this mini-course :

I. de Rham theorem, Poincaré and Alexander dualities in simplicial context, Čech-de Rham cohomology, relative de Rham theorem.

II. Localization and associated residue, Thom class, Grothendieck residue, singular holomorphic foliation.

III. Dolbeault theorem, relative Dolbeault cohomology, local cohomology.

IV. Atiyah class, Sato hyperfunction.

Čech-de Rham cohomology, particularly the relative version, combined with the Chern-Weil theory has been extensively used in the localization problem of characteristic classes. It started with the residue problem of singular holomorphic foliations and the theory was then transferred to the fixed point theory of discrete dynamics. The philosophy behind is rather simple. Namely, once we have some kind of vanishing theorem on the non-singular part, certain characteristic classes are localized at the set of singular points and the localization gives rise to residues and the residue theorem via the Alexander duality. The facility of computing the residues is another advantage of this method. The idea and the techniques turned out to be effective in many other problems, characteristic classes of singular varieties and localized intersection theory, to name a few.

A similar theory may be developed for the Dolbeault complex, the relevant characteristic classes in this case being the Atiyah classes. In particular, the relative Dolbeault cohomology turns out to be canonically isomorphic with the local (relative) cohomology of A. Grothendieck and M. Sato with coefficients in the sheaf of holomorphic forms. This gives a handy way of expressing the latter and would possibly lead to many applications. One of them is already apparent, i.e., we have a simple way of expressing the Sato hyperfunctions and some of the fundamental operations on them.

In this course, I will try to explain the basic ideas with some examples and applications.

References

1. M. Abate, F. Bracci and F. Tovena, *Index theorems for holomorphic self-maps*, Ann. of Math. **159** (2004), 819-864.
2. M. Abate, F. Bracci, T. Suwa and F. Tovena, *Localization of Atiyah classes*, Rev. Mat. Iberoam. **29** (2013), 547-578.
3. P. Baum and R. Bott, *Singularities of holomorphic foliations*, J. Differential Geom. **7** (1972), 279-342.
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5. J.-P. Brasselet, J. Seade and T. Suwa, *Vector Fields on Singular Varieties*, Lecture Notes in Math. **1987**, Springer 2009.

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8. D. Lehmann and T. Suwa, *Residues of holomorphic vector fields relative to singular invariant subvarieties*, J. Differential Geom. **42** (1995), 165-192.
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11. T. Suwa, *Residue Theoretical Approach to Intersection Theory*, Real and Complex Singularities, Contemporary Math., American Math. Soc. **459** (2008), 207-261.
12. T. Suwa, *Čech-Dolbeault cohomology and the $\bar{\partial}$-Thom class*, Advanced Studies in Pure Math. **56**, 2009, 321-340.

Residues, meromorphic connections and local holomorphic dynamics

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The aim of this mini-course is to introduce the audience to geometrical and cohomological techniques developed in the last fifteen years for studying the local holomorphic dynamics of a germ of holomorphic self-map in several complex variables. After recalling the notion of blow-up of a holomorphic germ at a fixed point, we shall show how to associate to a holomorphic self-map with an hypersurface S of fixed points a canonical singular foliation in Riemann surfaces and two partial meromorphic connections along the leaves of this foliation. Using the partial meromorphic connections and the Čech-de Rham theory developed by Lehmann and Suwa it is possible to prove several index (or residue) theorems having dynamical consequences. As a first application, we shall prove a 2-dimensional Leau-Fatou theorem, showing that all 2-dimensional holomorphic germs tangent to the identity with an isolated fixed point admit a parabolic curve (the analogue of a petal in the classical 1-dimensional Leau-Fatou flower theorem). Time permitting, we shall also show how to define geodesics of meromorphic connections and how to use them to study the dynamics of the time 1-map of 2-dimensional homogeneous vector fields.

References

1. M. Abate: *The residual index and the dynamics of holomorphic maps tangent to the identity.* Duke Math. J. **107** (2001), 173–207.
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3. M. Abate, F. Bracci, F. Tovena: *Index theorems for holomorphic maps and foliations.* Indiana Univ. Math. J. **57** (2008), 2999–3048.
4. M. Abate, F. Tovena: *Poincaré-Bendixson theorems for meromorphic connections and holomorphic homogeneous vector fields.* J. Diff. Eq. **251** (2011), 2612–2684.
5. M. Abate, F. Bianchi: *A Poincaré-Bendixson theorem for meromorphic connections on compact Riemann surfaces.* Math. Z. **282** (2016), 247–272.

25th (Tue)

Automorphisms of $\mathbb{C}^2$ with an invariant Fatou component biholomorphic to $\mathbb{C} \times \mathbb{C}^*$

Jasmin Raissy (Toulouse)
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Abstract : I will present the construction of a family of automorphisms of $\mathbb{C}^2$ having an invariant, non-recurrent Fatou component biholomorphic to $\mathbb{C} \times \mathbb{C}^*$ and which is attracting, in the sense that all the orbits converge to a fixed point on the boundary of the component. (This is a joint work with Filippo Bracci and Berit Stenoones).

Thom form in equivariant Čech-de Rham theory

Ko Fujisawa (Hokkaido, Graduate school)
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Abstract : In this talk, we introduce the foundation of a G-equivariant Cech-de Rham theory for a compact Lie group G by using the Cartan model of equivariant differential forms. Our approach is quite elementary without referring to the Mathai-Quillen framework. In particular, by direct computation, we give an explicit formula of the universal equivariant Thom form by localizing a certain equivariant Chern form, which deforms the classical Bochner-Martinelli kernel.

The escaping set of transcendental self-maps of the punctured plane

Davide Marti-Pete (Kyoto)
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Abstract : In this talk we study the iteration of holomorphic self-maps of C*, the complex plane with the origin removed, for which both zero and infinity are essential singularities. The escaping set of such maps consists of the points whose orbit accumulates to zero and/or infinity following what we call essential itineraries. We show that the Julia set always contains escaping points with every essential itinerary. The concept of essential itinerary leads to a partition of the escaping set into uncountably many disjoint sets, the boundary of each of which is the Julia set. Under certain hypotheses, each of these sets contains uncountably many curves to zero and infinity. We also use approximation theory to provide examples of functions with escaping Fatou components.

27th (Thu)

Local cohomology, Grothendieck local residues and algorithms

Shinichi Tajima (Tsukuba)
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Abstract : The theory of local cohomology is a cornerstone of commutative algebra, algebraic geometry and complex analysis. In this talk, we consider local cohomology, residues and duality in the contexts of singularity theory and of computational algebraic analysis. We first show that local cohomology togather with the Grothendieck local duality can be used to compute standard bases of zero-dimensional ideals in power series rings. The proposed method has various applications such as
(i) ideal quotient, (ii) ideal intersection, (iii) Tjurina stratification, (iv) Grothendieck local residue, (v) logarithmic vector fields, (vi) Bruce-Robers's Milnor number (vii) Samuel multiplicity, (viii) Teissier invariant μ^*, (ix) limiting tangent space (x) local Euler obstruction, (xi) generic Lê number, (xi) b-function.
We will describe in particular a method for computing Grothendieck local residues.

Intuitive representation of local cohomology groups

Daichi Komori (Hokkaido, Graduate school)
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Abstract : The theory of hyperfunctions was established by M. Sato with purely algebraic methods. It is based on the idea of sheaf cohomology, in particular, he define hyperfunctions by taking a local cohomology to the sheaf of holomorphic functions. As a consequence, it cannot be easily understood. In order to surmount this difficulty, A. Kaneko and M. Morimoto gave another definition of hyperfunctions. They realize a hyperfunction by taking sums of holomorphic functions. Thanks to their idea, the calculation of hyperfunctions became easier. Their idea can be applied not only a sheaf of holomorphic functions but also general sheaves. In this talk, to generalize their idea to local cohomology groups,we construct general framework which enables us to define their intuitive representation. As an application, we introduce Laplace hyperfuctions in several variables, which was defined by N. Honda and K. Umeta, and construct its intuitive representation.

This research is joint work with Kohei Umeta.

Several operations on Sato hyperfunctions from a viewpoint of Čech-Dolbeault cohomology

Naofumi Honda (Hokkaido)
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Abstract : Sato's hyperfunction is one of the most important objects in analysis as one can see its importance through the following example: For a linear ordinary differential equation with analytic coefficients like

$$\left(x^m \frac{d}{dx^n} + a_{n-1}(x)\frac{d}{dx^{n-1}} + \cdots + a_0(x)\right)u = 0$$

near the origin, it is well-known that the equation has $n + m$, i.e., the multiplicity of its leading symbol $x^m \xi^n$, linearly independent hyperfunction solutions at the origin. One can never expect, however, such a beautiful result for distribution solutions whose dimension is generally less than $n + m$ when the equation has irregular singularity at the origin.

In spite of its importance, a few people can understand the theory because a hyperfunction is defined by applying a local cohomology functor to the sheaf of holomorphic functions. To make the theory democratized, A. Kaneko and M. Morimoto invented so-called "intuitive representation" of a hyperfunction, by which we can finally avoid understanding a general theory of sheaf cohomology. Recently this method is generalized by D. Komori and it can be now applied to much wider class of hyperfunctions such as a Laplace hyperfunction.

On the other hand, T. Suwa proposes defining a hyperfunction by using his Čech-Dolbeault cohomology theory. This method makes several operations on a hyperfunction quite simple mainly because the cohomological residue map is given by the Dolbeault resolution, from which a lot of operations for a hyperfunction are induced. Furthermore, the boundary value morphism, which is considered to be the most important one in the hyperfunction world also works very well in this framework. I will give, in this talk, explicit description of several operations on a hyperfunction in terms of Čech-Dolbeault cohomology.

This is a joint work with Takeshi Izawa and Tatsuo Suwa.

28th (Fri)

The Riemann hypothesis and holomorphic index in complex dynamics

Tomoki Kawahira (Tokyo Inst. Tech.)
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Abstract : We give an interpretation of the Riemann hypothesis in terms of complex and topological dynamics. For example, the Riemann hypothesis is affirmative and all zeros of the Riemann zeta function are simple if and only if a certain meromorphic function has no attracting fixed point, or equivalently, there is no topological disk whose image by this meromorphic function is compactly contained by itself. To obtain this, we use holomorphic index (residue fixed point index), which characterizes local properties of fixed points in complex dynamics.

Fixed points of some polynomial automorphisms of $\mathbb{C}^n$

Tetsuo Ueda (Kyoto)
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Abstract : We will investigate fixed point indices for a class of polynomial automorphism of $\mathbb{C}^n$ as an analogue to the class of compositions of generalized Hénon maps for $\mathbb{C}^2$.

Dynamics of families of maps tangent to the identity

Marco Abate (Pisa)
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Abstract : We shall show how it is possible to associate to any holomorphic germ f tangent to the identity in several complex variables a singular holomorphic foliation in Riemann surfaces of a projective space and two meromorphic connections along the foliation, sharing the same (real) geodesics. When f is the time-1 map of a homogeneous vector field, this geodesic flow allows to recover completely the dynamics of the original germ, and can be studied geometrically. In particular, we shall present local normal forms around the singularities and a global Poincare-Bendixson-like theorem giving the asymptotic behaviour of geodesics. As a consequence we shall be able to describe the dynamics of some interesting families of germs tangent to the identity in two complex variables in a full neighbourhood of the origin. (Joint work with F. Tovena and F. Bianchi).

Characteristic classes for infinitesimal deformations of foliations

Taro Asuke (Tokyo)
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Abstract : Given a smooth one-parameter family of foliations, we can define the derivative of characteristic classes such as the Godbillon–Vey class. Derivatives are defined not only for actual families but for infinitesimal deformations. In the latter case, derivatives are constructed by means of connections and their infinitesimal deformations. On the other hand, associated with infinitesimal deformations, some characteristic classes of another kind, e.g. Fuks–Lodder–Kotschick class, are defined. In this talk, I will present a framework by which these classes appear in a unified manner. This will be done by means of a certain extension of the normal bundle of the foliation which is called 2-normal bundle, of a natural connection associated with the foliation and infinitesimal deformations, and the Chern–Weil construction. The details can be found in [2]. Related topics on the Godbillon–Vey class can be found in [1] and [3].

References

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[3] T. Asuke, *Notes on 'Infinitesimal derivative of the Bott class and the Schwarzian derivatives'*, Tohoku Math. J. **69** (2017), 129–139.

Relative Bott-Chern cohomology

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Abstract : We present a theory of relative versions of Bott-Chern cohomology and its triplet siblings together with some examples of refined residues. A joint work with M. Correa. Reference: https://arxiv.org/abs/1705.09420