

Research Report of JSPS-ITP  
(The international sending-elevating project for young mathematicians  
based on singularity, topology and mathematical analysis: Hokudai model)

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Research Report:

Thanks to the support from the ITP project in Hokkaido University, I visited Durham, England from October 1 to December 1 to study the differential geometry of hypersurfaces in the Euclidean sphere.

I study the differential geometry of submanifolds in Euclidean  $n$ -sphere as application of singularity theory. Mainly I was studying about the singularity theory of families of orthogonal projections of surfaces in the Euclidean 3-sphere to great spheres. Singularities of these projections related to the asymptotic directions of surfaces. Projections of surfaces in the Euclidean and projective 3-space are well studied, and in the hyperbolic space case is researched by Shyuishi Izumiya and Farid Tari. I am interested in analogous study for embedded surfaces in the Euclidean 3-sphere.

Professor Tari was my supervisor. He is one of the masters of singularity theory. Professor Tari and I had weekly meetings. He gave me many important advises and suggestions about study of orthogonal projections. Under the guidance of him, I studied about geometric characterizations of the generic singularities of the orthogonal projections of surface in the Euclidean 3-sphere to great sphere. This orthogonal projections can be considered locally as a map germ from Euclidean 2-space to same space around the origin. Singularities of these map germs are well studied. I studied the conditions for the singularities of orthogonal projections to  $A$ -equivalent locally to generic singularities of these map germs.

He also gave me great care about my life in Durham. I would like to thank him very much.

Professor Tari gave me a chance to talk my study at the Geometry seminar. In this seminar, I talked about the evolute of hypersurfaces in the Euclidean sphere as an application of Lagrangian singularity theory. The title of my talk

was “The evolute of hypersurface in Euclidean  $n$ -sphere”.

I have been studying the evolute of hypersurface in Euclidean  $n$ -sphere since four years ago. The evolute is denoted by caustics of a Lagrangian immersion to the symplectic manifold which is given by the symplectification of the contact manifold in the product of the  $n$ -spheres which is the well-known spherical Legendrian duality. Then the classification of singularities of evolute can be characterised used by the idea of Lagrangian equivalence of Lagrangian singularities. Geometric meanings of this classification are given by using theory of contact with foliations that was considered by Montaldi. After I did this talk, I got some valuable advices to study about meanings of each singularity of evolute that was classified.

Finally, I wish to express gratitude to ITP which let me have this valuable experience. Thank you very much for ITP support.