Dyadic Hausdorff Capacities and Their Applications to Function Spaces

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Abstract

It is well-known that the Hausdorff capacity and its dyadic version play an important role in potential theory, harmonic analysis and the modern theory of partial differential equations; see, for example, [1, 2, 3, 5, 7, 9, 11].

In this talk, we will give a counterexample to show that the classical dyadic Hausdorff capacity $\widetilde{H}^d_\infty$ on $\mathbb{R}^n$ when $n \geq 2$ and $0 < d \leq n-1$ is not a capacity in the sense of Choquet. Motivated by Carleson [4], we introduce a variant of the classical dyadic Hausdorff capacity, $\widetilde{H}^d_{\infty,0}$, and we further prove that $\widetilde{H}^d_{\infty,0}$ is a capacity in the sense of Choquet.

On the other hand, we introduce a new class of function spaces $\dot{F}^{s,\tau}_{p,q}(\mathbb{R}^n)$, which unify and generalize the Triebel-Lizorkin spaces with both $p < \infty$ ([10]) and $p = \infty$ ([10, 8]) and Q spaces ([6]) on $\mathbb{R}^n$. Moreover, using dyadic Hausdorff capacities, we introduce some Hardy-Hausdorff spaces, and prove that their dual spaces are just the spaces $\dot{F}^{s,\tau}_{p,q}(\mathbb{R}^n)$. Our results establish relationships between Triebel-Lizorkin spaces and Q spaces, which answers a question posed in [5].

References

[1] David R. Adams, A note on the Choquet integrals with respect to Hausdorff capacity, in: Function spaces and applications, Proc. Lund 1986, Lecture Notes in Math. 1302 (Springer, Berlin, 1988), 115-124.

[2] David R. Adams and L. I. Hedberg, Function Spaces and Potential Theory, Springer-Verlag, Berlin, 1996.

[3] David R. Adams, Choquet integrals in potential theory, Publ. Mat. 42(1998), 3-66.

[4] L. Carleson, Selected Problems in Exceptional Sets, Van Norstrand, Princeton, 1967.

[5] G. Dafni and J. Xiao, Some new tent spaces and duality theorems for fractional Carleson measures and $Q_\alpha(\mathbb{R}^n)$, J. Funct. Anal. 208(2004), 377-422.

[6] M. Essén, S. Janson, L. Peng and J. Xiao, Q spaces of several variables,Indiana Univ. Math. J. 49(2000), 575-615.

[7] R. Fefferman, A theory of entropy in fourier analysis, Adv. Math. 30(1978), 171-201.

[8] M. Frazier and B. Jawerth, A discrete transform and decompositions of distribution spaces, J. Funct. Anal. 93(1990), 34-171.

[9] J. Orobitg and J. Verdera, Choquet integrals, Hausdorff content and the Hardy-Littlewood maximal operator, Bull. London Math. Soc. 30(1998), 145-150.

[10] H. Triebel, Theory of function spaces, Birkhäuser Verlag, Basel, 1983.

[11] J. Xiao, Sharp isoperimetric and Brunn-Minkowski estimates for Hausdorff capacity, Contemp. Math. 411(2006), 141-143.