Two Problems Associated to Schrödinger operator

Lizhong Peng

Peking University

Abstract: In this talk, we consider two problems about Schrödinger Operator with some special potential. One is L^p boundedness of some commutator operators associated with Schrödinger operator $P = -\Delta + V(x)$ on $\mathbb{R}^n$, $n \geq 3$. We assume that $V(x)$ is non-zero, nonnegative, and belongs to B_q for some $q > n/2$. Let $T_1 = (-\Delta + V)^{-1}V$, $T_2 = (-\Delta + V)^{-1/2}V^{1/2}$, and $T_3 = (-\Delta+V)^{-1/2}\nabla$. We obtain that $[b, T_j]$ $(j = 1, 2, 3)$ is a bounded operator on $L^p(\mathbb{R}^n)$ when p ranges in a interval, where $b \in \mathbf{BMO}(\mathbb{R}^n)$. We also discuss the converse problem.

The other problem is about the decomposition of $H^1_L \times BMO_L$.We prove that, for $f \in H^1_L, b \in BMO_L$, the point-wise product $b \cdot f$ as a Schwartz distribution, denoted by $b \times f \in S'(R^n)$, can be decomposed in two parts, that is $b \times f = u + v$ where $u \in L^1(R^n)$ while v lies in $Hardy-Orlicz$ space associated with Schrödinger operators $H^{\mathcal{P}}_L(R^n, d\mu)$.

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