Preduals of generalized Morrey-Campanato spaces

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Let $X = (X, d, \mu)$ be a space of homogeneous type. Using atoms, Coifman and Weiss [1] defined the Hardy space $H^p(X)$ as the subspace of the dual of $\mathrm{Lip}_\alpha(X)$ and they proved that $\mathrm{Lip}_\alpha(X)$ is the dual of $H^p(X)$. Their results are generalization of the case $X = \mathbb{R}^n$. We note that, in [1], $\mathrm{Lip}_\alpha(X)$ was regarded the space of functions modulo constants. Therefore, we denote the fact above by $(H^p(X))^* = \mathrm{Lip}(X)/\mathcal{C}$. Let $\mathcal{L}_{p,\phi}(X)$ be the Campanato space which is a genaralization of $\mathrm{Lip}_\alpha(X)$. In this talk we define a generalized Hardy space $H_U^{\Phi,q}(X)$ as the subspace of the dual of $\mathcal{L}_{p,\phi}(X)/\mathcal{C}$ and prove that $\mathcal{L}_{p,\phi}(X)/\mathcal{C}$ is the dual of $H_U^{\Phi,q}(X)$, i.e. $\left(H_U^{\Phi,q}(X)\right)^* = \mathcal{L}_{p,\phi}(X)/\mathcal{C}$. The definition of $H^p(X)$ in [1] is a special case of ours. We note that the predual of $\mathcal{L}_{p,\phi}(X)/\mathcal{C}$ is not unique. Zorko [6] defined another predual of $\mathcal{L}_{p,\phi}(X)/\mathcal{C}$ in the case $X = \mathbb{R}^n$. Our definition is a generalization of both definitions.

We also define a space $B_U^{\Phi,q}(X)$ generated by blocks (”block” means an atom without the cancellation property), and prove that the dual of $B_U^{\Phi,q}(X)$ is a Morrey space $L_{p,\phi}(X)$. This result is a genaralization of Long [2]. If $X = \mathbb{R}^n$, $d(x, y) = |x-y|$, μ is Lebesgue measure, $\Phi(r) = r$ and $U(r) = r(1+\log^+(1/r))$, then $B_U^{\Phi,q}(X)$ is the space introduced by Taibleson and Weiss [5] and Lu, Taibleson and Weiss [3].

It is known that $\mathcal{L}_{p,\phi}(X)/\mathcal{C}$ and $L_{p,\phi}(X)$ coincide under a cartain condition ([4]). We show that $H_U^{\Phi,q}(X)$ and $B_U^{\Phi,q}(X)$ coincide under the correspondent condition.

References

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