Representation Theory, Systems of Differential Equations and their Related Topis

July 2 – 6, 2007, Hokkaido University

Abstracts

Monday, July 2

Toshio Oshima (University of Tokyo)

Heckman-Opdam hypergeometric functions and their confluences

Among the completely integrable quantum systems associated with root systems, Heckman-Opdam hypergeometric systems and their confluences are considered to be a good class of "rigid local systems". We study suitable limits, such as confluence of singularities and specialization with respect to singular sets or parameters, which enable us to get some connection formula. This is a result of a joint work with N. Shimeno.

Taku Ishii (Chiba Institute of Technology)

Explicit formulas of Whittaker functions on real semisimple Lie groups

In this talk we will explain recent progress on the explicit formulas of Whittaker functions on real semisimple Lie groups developed by the speaker and his collaborators. We mainly discuss Whittaker functions related to the principal series representations of $SL(n, \mathbb{R})$ and $SO(2n+1, \mathbb{R})$, and mention some applications to automorphic forms.

Birne Binegar (Oklahoma State University)

Whittaker vectors, generalized hypergeometric functions and a matrix calculus

Motivated by some open problems in analytic representation theory, we derive certain holonomic systems of PDEs of matrix argument corresponding to smooth Whittaker vectors for spherical principal series representations of $SL(2n, \mathbb{R})$. A novel matrix calculus is then introduced and developed. This matrix calculus not only allows us to view the Whittaker PDEs as a particularly simple and natural generalization of the classical confluent hypergeometric equation, it also allows us to bring to bear a number of new, but seemingly very classical, invariant theory identities to the problem of constructing and analyzing of explicit solutions of the Whittaker PDEs via a generalized power series technique. A particular advantage of the matrix calculus method is the explicit, in fact manifest, determination of the behavior of solutions as they approach the singular loci corresponding to the non-open L-orbits in $\overline{\mathfrak{n}}$.

Tuesday, July 3

Nobuki Takayama (Kobe University)

Modified $\mathcal{A}$-hypergeometric system

We consider a definite integral associated to a $d \times n$ matrix A with integer elements, n-vector w of integers, and a d-vector β of complex numbers. The integral is

$$F(\beta, x, t) = \int_C \exp\left(\sum_{i=1}^{n} x_i t^{w_i} s^{a_i}\right) s^{-\beta-1} ds,$$
$$s = (s_1, \ldots, s_d),\ \beta = (\beta_1, \ldots, \beta_d),$$

where a_i is the i-th column vector of the matrix A.

We call it a *modified $\mathcal{A}$-hypergeometric integral* and a system of differential equations satisfied by this integral is called a *modified $\mathcal{A}$-hypergeometric system.*

This is a variation of the $\mathcal{A}$-hypergeometric system (GKZ system), which has been enthusiastically studied by some mathematicians with (computational) commutatitive algebra, the D-module theory, polyhedral combinatorics, and analysis since the work of Gel'fand, Kapranov, Zelevinski in 1987.

We will introduce this new modified system and discuss some fundamental properties and also show some experimental results on computers on this new system.

Uli Walther (Purdue University)

Regularity and slopes of hypergeometric systems along linear subspaces

We study the irregularity sheaves attached to the A-hypergeometric D-module $M_A(\beta)$ introduced by Gel'fand et al., where $A \in \mathbb{Z}^{d\times n}$ is pointed of full rank and $\beta \in \mathbb{C}^d$. More precisely, we investigate the slopes of this module along coordinate subspaces. In the process we describe the associated graded ring to a positive semigroup ring for a filtration defined by an arbitrary weight vector L on torus invariant generators. To this end we introduce the (A, L)-umbrella, a simplicial complex determined by A and L, and identify its facets with the components of the associated graded ring. We then etablish a correspondence between the full (A, L)-umbrella and the components of the L-characteristic variety of the A-hypergeometric D-module. We compute in combinatorial terms the multiplicities of these components in the L-characteristic cycle of the associated Euler-Koszul complex, identifying them with certain intersection multiplicities. We deduce from this that slopes of $M_A(\beta)$ are combinatorial, independent of β, and in one-to-one correspondence with jumps of the (A, L)-umbrella. This confirms a conjecture of Sturmfels and gives a converse of a theorem of Hotta: $M_A(\beta)$ is regular if and only if A defines a projective variety. This is a joint work with Mathias Schulze from Oklahoma State University.

Hironobu Kimura (Kumamoto University)

The general Schlesinger system and Ward correspondence

Following the idea of Mason and Woodhouse, we understand the Schlesinger system and its degenerated systems as defined on the Grassmannian manifold $Gr(2, N+1)$ in connection with the generalized anti self dual Yang Mills equation. Using this point of view we discuss a construction of particular solutions of the Schlesinger system.

Yoshishige Haraoka (Kumamoto University)

Studies on regular holonomic systems from the viewpoint of rigidity

Rigidity argument for Fuchsian ordinary differential equations brings many remarkable results – we have algorithms for constructing all rigid equations, and see that any rigid equation has integral representations of solutions, which enables us to discribe connection coefficients in terms of integrals. In this talk we want to discuss how rigidity viewpoint for regular holonomic systems works, and shall show several examples concerning non-rigid ordinary differential equations.

Jiro Sekiguchi (Tokyo University of Agriculture and Technology)

Singular curves, Saito free divisors and root systems

My original interest is to find out examples of Saito free divisors by constructing Lie algebras generated by logarithmic vector fields along them. I will explain an idea to construct Saito free divisors in a three dimensional affine space related with 1-parameter deformation space of singular curves having simple singularities or some of 14 exceptional families of isolated singularities in the sense of V. I. Arnol'd. Then I discuss a relationship between such divisors and root systems of isolated singularities.

Wednesday, July 4

Gen Mano (University of Tokyo)

The unitary inversion operator for the minimal representation of the indefinite orthogonal group $O(p, q)$

The indefinite orthogonal group $O(p, q)$ ($p + q$ even, greater than four) has a distinguished infinite dimensional irreducible unitary representation called the 'minimal representation'. Among various models, the L^2-model of the minimal representation of $O(p, q)$ was established by Kobayashi-Ørsted (2003). In this talk, we present an explicit formula for the unitary inversion operator, which plays a key role for the analysis on this L^2-model. Our proof uses the Radon transform of distributions supported on the light cone. This is a joint work with T. Kobayashi.

Tomohide Terasoma (University of Tokyo)

Beilinson regulator and bar complex for Deligne cohomology

We describe a homomoprhism of coalgebras for the realization functor to the category of mixed Hodge complexes to that defines the mixed Hodge structures. Using recovering principle, we show that this map gives a Beilinson-regulator .

Ken-ichi Sugiyama (Chiba University)

A geometric analog of the Iwasawa conjecture

We will compare the leading terms of the Ruelle L-function and the Alexander invariant for a complete hyperbolic threefold of finite volume. Our result may be regarded as a geometric analog of the Iwasawa conjecture in the number theory.

Thursday, July 5

Yutaka Matsui (University of Tokyo)

Lefschetz fixed point formulas over singular varieties (a joint work with K.Takeuchi)

In this talk, we shall introduce our recent results on Lefschetz fixed point formulas. As a byproduct, we obtain some explicit descriptions of Lefschetz numbers for the cohomology groups of singular varieties. Several examples concerning Schubert and toric varieties etc. will be also presented.

Ryohei Hattori (Hokkaido University)

On periods of cyclic triple coverings of the complex projective line and theta constants

We consider the moduli space Λ of cyclic triple coverings of the complex projective line branching at $3n$ points. By using $N = (3n)!/(3!(n!)^3)$ polynomials, we give an embedding $\imath$ of Λ into the complex projective space of dimension $N-1$. We construct the period map per from Λ to the Siegel upper half space. We give a map Θ by the ratio of the cubes of N theta constants so that $\imath = \Theta \circ \mathrm{per}$ holds.

Our results are analogies to Thomae's formulas for families of hyperelliptic curves.

William Traves (United States Naval Academy)

Differential operators and invariant theory

Whenever a group acts on an algebraic variety X, the ring of invariant functions $\mathbb{C}[X]^G$ determines the Geometric Invariant Theory quotient variety. I'll explain this procedure and then discuss two rings of differential operators associated to the group action: one of these is the ring of invariant differential operators on X, the other is the ring of differential operators on the quotient variety. I'll describe the subtle relationship between these two rings and describe how they can be computed in the case when the quotient variety is the Grassmannian of lines in projective 3-space.

Kazufumi Kimoto (University of the Ryukyus)

Invariant theory of singular α-determinants

The α-determinant is a certain polynomial function having a parameter α, which can be regarded as a one-parameter interpolation of the ordinary determinant and permanent. When α is equal to a reciprocal of a negative integer whose absolute value does not exceed the matrix size, the α-determinants obtain interesting properties so that we can construct relative GL-invariants by using them. In the talk, we recall the basic properties of α-determinants, explain the construction of relative GL-invariatns and discuss several related topics. (Joint work with Masato Wakayama (Kyushu University))

Minoru Itoh (Kagoshima University)

Schur type functions associated to polynomial sequences of binomial type

We introduce Schur type functions associated to polynomial sequences of binomial type. These can be regarded as a generalization of the usual Schur functions and the factorial shifted Schur functions. Using these functions, we can describe the eigenvalues of some central elements of the universal enveloping algebras of classical Lie algebras.

Friday, July 6

Salem Ben Saïd (Université Henri Poincaré)

On the role of $SL(2,\mathbb{R})$ in the theory of Dunkl operators

In a joint papers with B. Ørsted, we showed that there exists an infinitesimal representation of $\mathfrak{sl}(2,\mathbb{R})$, on the Schwartz space on $\mathbb{R}^N$, that can be used to study several topics in the theory of Dunkl operators. This representation can be thought of as analogue of the classical infinitesimal metaplectic representation of $\mathfrak{sl}(2,\mathbb{R})$. In this talk we will discuss the integrability of the infinitesimal representation to the universal covering group $\widetilde{SL(2,\mathbf{R})}$, and then we shall describe the role of the representations theory in proving: Huygens' principle for the wave equation for the Dunkl Laplacian operator, and the Bochner identity for the Dunkl transform. If time permits we will also describe a Harish-Chandra restriction type theorem for the Dunkl transform.

Atsushi Wakamiko (Hokkaido University)

On the freeness of multi-Coxeter arrangements of type B_ℓ and F_4

Let W be the finite irreducible reflection group of type B_ℓ ($\ell \geq 2$) or F_4. Let $\mathcal{A} = \mathcal{A}(W)$ be the corresponding Coxeter arrangement and let $m\colon \mathcal{A} \longrightarrow \mathbb{Z}_{\geq 0}$ be a multiplicity which is constant both on the "long roots part" and on the "short roots part". In this talk, we give the exponents of the multiarrangement $(\mathcal{A}, m)$ and construct a basis for the module $D(\mathcal{A}, m)$ of all vector fields which contact to each reflecting hyperplanes $H \in \mathcal{A}$ with multiplicities $m(H)$. This is a joint work with Hiroaki Terao.

Jing-Song Huang (Hong-Kong University of Science and Technology)

Dirac operators and Lie algebra cohomology

We use Kostant cubic Dirac operator to define Dirac cohomology and show that it is closely related to the nilpotent Lie algebra cohomology. We also show that Dirac cohomology has nice properties for studying unitary representations of semisimple Lie groups and it is more accessible for calculation. This is joint work with P. Pandzic and D. Renard.