The NORThern Workshop on Representation Theory of Lie Groups and Lie Algebras

Dates: March 6–9, 2007

Venue: 8-309 at Department of Mathematics, Hokkaido University (on the 3rd floor of Science Bldg #8)

Akihito Wachi (Hokkaido Institute of Technology) March 6, 13:45 - 14:45

Title: The strong Lefschetz property of the cohomology rings of the flag varieties

Abstract: The hard Lefschetz theorem for cohomology rings of non-singular varieties describes the graded structure as follows. Let $A_i = H^{2i}(X)$ be the $2i$-th cohomology of a non-singular variety X, and c the maximum integer satisfying $A_c \neq 0$. Then there exists an element $l \in A_1$, such that the multiplication map $\times l^{c-2i} : A_i \to A_{c-i}$ is bijective for any integer i $(0 \leq i \leq c/2)$. In this case, the graded ring $A = A_0 + A_1 + \cdots + A_c$ is said to have the strong Lefschetz property, and l is called a strong Lefschetz element.

In this talk, (1) we determine the set of the strong Lefschetz elements for flag varieties, (2) give the necessary condition for the strong Lefschetz elements for partial flag varieties, (3) and generalize these results to non-crystallographic cases H_3, H_4 and $\mathrm{I}_2(m)$.

This is a joint work with Toshiaki Maeno and Yasuhide Numata.

Kyo Nishiyama (Kyoto University) March 6, 15:00 - 16:00

Title: Degenerate principal series and the geometry of null fiber (joint work with Peter Trapa)

Abstract: Let us consider a reductive dual pair in the large symplectic group. Then there is a double fibration by moment maps from a (complexified) Lagrangian subspace to the tangent spaces of the associated Riemannian symmetric spaces. We study the null fiber of the moment map and the relationship with degenerate principal series.

Hiroshi Oda (Takushoku University) March 6, 16:30 - 17:30

Title: Eigenvalues of contracted invariants

Abstract: For a non-compact irreducible Riemannian symmetric space X let $G' = I(X)$ (the group of the isometries) and $G = I_0(X)$ (the identity component). Thus $X = G/K$ for a maximal compact subgroup K of G. Let $\mathbf{D}$ (resp. $\mathbf{D}'$) be the ring of the G-invariant (resp. G'-invariant) differential operators on X. For many X, we explicitly construct a generator system of $\mathbf{D}$ or $\mathbf{D}'$ by the contraction of small-dimensional K-types. The eigenvalues of the generators are calculated by the generalized Harish-Chandra homomorphism. In the Hermitian case, such generator system is known as the Shimura system and the eigenvalues of the generators were recently calculated by G. Zhang by a different method.

Hiro-aki Narita (Osaka City University) March 7, 9:15 - 10:15

Title: Generalized Whittaker functions on Sp(1,q) for quaternionic discrete series and their applications to automorphic forms

Abstract: A fundamental tool to study automorphic forms is a Fourier expansion. In Fourier expansions of automorphic forms there naturally appear generalized Whittaker functions for admissible representations. In this talk we consider the case of quaternionic discrete series representations of the non-split group Sp(1,q). For this case we present an explicit formula for the generalized Whittaker functions and the multiplicity formula for the generalized Whittaker models. We then discuss their applications to automorphic forms, e.g. the Fourier expansion and explicit constructions of automorphic forms etc.

Jing-Song Huang (Hong Kong University of Science and Technology)
March 7, 10:45 - 11:45

Title: Dirac cohomology, Lie algebra cohomology and branching rules

Abstract: A fundamental problem in representation theory is to understand the decomposition of an irreducible unitary representation of a real reductive Lie group when it is restricted to a closed reductive subgroup. The aim of this talk is to show how to use Dirac cohomology and Lie algebra cohomology to obtain and prove branching rules for some of the most interesting unitary representations.

Leticia Barchini (Oklahoma State University) March 7, 13:45 - 14:45

Title: Semi-Invariant Systems of Differential Operators on Manifolds

Abstract: Kostant's theory of conformally invariant differential equations on certain homogeneous manifolds is generalized to cover semi-invariant systems of differential operators acting on spaces of smooth sections of vector bundles over manifolds. We explore the relation between this generalized theory and the theory of intertwining operators.

As a special case, we consider G a reductive group with parabolic subalgebra $P = LN$ and we take $M = N \cdot o \subset G/\bar{P}$. An irreducible representation, E, of L, determines a vector bundle over M. We study the relation between semi-invariant systems of differential operators acting on $\Gamma^{\infty}(N \cdot o, E)$ and leading weight spaces of the generalized Verma module $\mathcal{U}(\mathfrak{g}) \otimes_{\mathcal{U}(\bar{\mathfrak{p}})} E^*$. In this setting, Huang partially generalized Kostant's theory. Huang abandoned the framework of conformally invariance in favor of that of differential intertwining operators.

This talk is based on joint work with Anthony Kable and Roger Zierau.

Kyo Nishiyama (Kyoto University) March 7, 15:00 - 16:00

Title: Asymptotic cone of semisimple orbits for symmetric pair

Abstract: Asymptotic cone is a device introduced by Borho and Kraft to analyze a deformation of orbits. In the case of the adjoint action of reductive groups, the asymptotic cone of any orbit is a union of nilpotent orbits.

We survey known facts about asymptotic cone and the relation to the cotangent bundle of a partial flag variety, then give a naive observation on a generalization to the case of symmetric pairs. In this case, conormal bundles over closed K_C-orbits on a partial flag variety play an important role, and the collection of such orbits are related to real parabolic induction.

This talk is closely related to the work of Peter Trapa on Richardson orbits for symmetric pairs (J. Alg. 286(2005)). In his paper, Richardson orbits are characterized as the associated varieties of derived functor modules.

Soo Teck Lee (National University of Singapore) March 7, 16:30 - 17:30

Title: A proof of the Littlewood-Richardson rule

Abstract: The algebra of polynomial functions on the space of $n \times (k + \ell)$ complex matrices carries an action by $\mathrm{GL}_n \times \mathrm{GL}_k \times \mathrm{GL}_\ell$. Its subalgebra of highest weight vectors can be used to study tensor products of GL_n representations, so it is called a GL_n tensor product algebra. In this talk, we will use this algebra to construct a proof of the the Littlewood-Richardson rule. This is joint work with Roger Howe.

Kazunari Sugiyama (University of Tsukuba) March 8, 9:15 - 10:15

Title: Decomposition formula for b-functions and its application

Abstract: We show that b-functions of a reducible prehomogeneous vector space have decompositions correlated to the decomposition of the representation if a certain multiplicity one property holds. As an application of the decomposition formula, we can determine the b-functions of prehomogeneous vector spaces of parabolic type arising from the special linear Lie algebras. This talk is based on joint work with Fumihiro Sato (Internat. J. Math. 17 (2006)).

Peter Trapa (University of Utah) March 8, 10:45 - 11:45
March 9, 13:00 - 14:00

Title: On the geometry of partial flag varieties

Abstract: Fix a complex reductive group G with Lie algebra $\mathfrak{g}$ and fix a nilpotent orbit in $\mathfrak{g}$. Let $\{e, f, h\}$ denote the corresponding Jacobson-Morozov triple, and let $\chi = \frac{1}{2}h$. Then the centralizer of χ in G acts with finitely many orbits on the eigenspaces of $\mathrm{ad}(\chi)$. The fundamental work of Lusztig and Kazhdan-Lusztig reveals, roughly speaking, how the geometry of these orbit closures is related to the character theory of unipotent representations of reductive p-adic groups. In these talks, we show (in favorable cases) that this geometry can be recovered from the structure of $K_{\mathbb{C}}$ orbits on a partial flag variety for $\mathfrak{g}$. Here $K_{\mathbb{C}}$ is the complexification of a maximal compact subgroup of a real form of G.

Leticia Barchini (Oklahoma State University) March 8, 13:45 - 14:45

Title: Semi-Invariant Systems of Differential Equations and Prehomogeneous Vector Spaces of Heisenberg Type

Abstract: Let G be a split semisimple real Lie group whose complexified Lie algebra $\mathfrak{g}$ is simple of rank greater than one. It is known that $\mathfrak{g}$ contains a unique conjugacy class of parabolic subalgebras of Heisenberg type. Let $\mathfrak{q}$ be one such parabolic subalgebra and write $\mathfrak{n}$ for the two step nilpotent nilradical. Decompose $\mathfrak{n} = V^+ \oplus Z(\mathfrak{n})$. If Q is the connected parabolic subgroup with Lie algebra $\mathfrak{q}$, then $Q = LN$ and the triple (L, Ad, V^+) is a prehomogeneous space. We say that prehomogeneous vector spaces constructed this way are of Heisenberg type.

We use invariant theory associated to (L, Ad, V^+), in particular the covariants, to build semi-invariant systems of differential equations. Each system is conformally invariant under the given Lie algebra. The system so constructed yield reducibility results for a family of scalar generalized Verma modules describing explicit homomorphisms between certain generalized Verma modules.

This talk is based on joint work with Anthony Kable and Roger Zierau.

Hisayosi Matumoto (University of Tokyo) March 8, 15:00 - 16:00
March 9, 14:15 - 15:15

Title: On irreducibility of the space of continuous Whittaker vectors

Abstract: Let G be a real semisimple Lie group and let $G = KAN$ be its Iwasawa decomposition. The Kostant-Lynch theory tells us the space of continuous Whittaker vectors on an irreducible G-representation with respect to a generic unitary character of N has a structure of a module over the finite W-algebra. As a generalization of the multiplicity one theorem for quasi-split groups, one may ask whether the space of continuous Whittaker vectors is irreducible with respect to the action of the finite W-algebra. In this talk, we discuss affirmative results including the case of type A non quasi-split groups.

Toshio Oshima (University of Tokyo) March 8, 16:30 - 17:30

Title: Root subsystems of a root system

Abstract: We classify isomorphic classes of the homomorphisms of a root system Ξ to a root system Σ which do not change Cartan integers. We examine several types of isomorphic classes defined by the Weyl group of Σ, that of Ξ and the automorphisms of Σ or Ξ etc. We introduce the concept of the dual pair for root systems which helps to study the action of the outer automorphism of Ξ on the homomorphisms.

Hideaki Morita (Oyama National College of Technology)
March 9, 9:15 - 10:15

Title: A combinatorial property of Garsia-Haiman modules and Green polynomials with two variables

Abstract: We study a combinatorial property, consistency of dimensions, of Garsia-Haiman modules for the symmetric groups. The property is conjectured to be rephrased in terms of representation theory, which is deeply related with the Green polynomials with two variables at roots of unity. In this talk, we give a result for a special case where the corresponding partitions are hooks.

Takeshi Hirai (Kyoto University) March 9, 10:45 - 11:45

Title: Characters of projective (= spin) factor representations of the infinite symmetric group

Abstract: Let $\mathfrak{S}_n$ be the n-th symmetric group, then it has two kinds of universal covering groups $\mathfrak{T}_n, \mathfrak{T}'_n$ for $n \geq 4, n \neq 6$. They are given by a system of generators and relations, and a unitary representation of $\mathfrak{T}_n$ or $\mathfrak{T}'_n$ is called a *projective* (or *spin*) representation of $\mathfrak{S}_n$ if it cannot be reduced to a representation of $\mathfrak{S}_n$. When we treat projective representations, there are no essential difference between $\mathfrak{T}_n$ and $\mathfrak{T}'_n$.

Here we treat the infinite version $\mathfrak{T}_\infty$ of $\mathfrak{T}_n$, and study its unitary representations which cannot be reduced to $\mathfrak{S}_\infty$ (called also *projective* or *spin*). Our aim is to obtain characters of projective factor representations of finite type. Such a character is given canonically as an *extremal* invariant positive definite function on $\mathfrak{T}_\infty$, when it is normalized. (This is a part of jointworks with A. Hora and E. Hirai)

Hiroyuki Ochiai (Nagoya University) March 9, 15:30 - 16:30

Title: TBA